Number System

Unit Digit:

Concept	Explanation	Example
Unit Digit	The digit in the one's place of a number.	Unit digit of **237** is **7.**
Unit Digit of Powers	Follows a cyclic pattern depending on the base.	Unit digit of 21=22^1 = 221=2, 22=42^2 = 422=4, 23=82^3 = 823=8...
Cyclic Patterns	Most digits (1–9) have repeating cycles in their powers.	2 → {2,4,8,6} (Cycle length = 4)
Shortcut Method	Divide power by cycle length & find remainder → use remainder to find unit digit.	Find unit digit of 7457^{45}745: Cycle of 7 = {7,9,3,1}, 45 mod 4 = 1 → Ans: **7**
Special Cases	Some numbers (like 5, 6) always end in the same digit for all powers.	5n5^n5n ends in **5**, 6n6^n6n ends in **6.**

Factors:

Concept	Explanation	Example
Factor/Divisor	A number that divides another number exactly (no remainder).	Factors of **12**: 1, 2, 3, 4, 6, 12
Total No. of Factors	Use prime factorization: Add 1 to each exponent and multiply.	12=22×3112 = 2^2 × 3^112=22×31 → (2+1)(1+1) = **6** factors
Sum of Factors	Use formula with prime factorization.	12=22×3112 = 2^2 × 3^112=22×31 → (1+2+4)(1+3) = 7×4 = **28**
Even/Odd Factors	Even → include only factors with at least one 2; Odd → exclude all 2s.	Odd factors of 36 = factors of 32=1,3,93^2 = 1, 3, 932=1,3,9
Perfect Square Check	A number is a perfect square if all exponents in its prime factorization are even.	36=22×3236 = 2^2 × 3^236=22×32 → Perfect square ✅
Co-prime Numbers	Two numbers having only 1 as their common factor.	(8, 15) are co-prime → GCD = 1
Highest Common Factor (HCF)	Greatest number that divides both numbers.	HCF of 12 and 18 = **6**
Least Common Multiple (LCM)	Smallest number divisible by both numbers.	LCM of 4 and 6 = **12**

No of Zeros:

Concept	Formula / Rule	Explanation / Example
Trailing zeros in $n!$	$\lfloor \frac{n}{5} \rfloor + \lfloor \frac{n}{25} \rfloor + \lfloor \frac{n}{125} \rfloor + \dots$	Count of 5s in prime factorization of $n!$; 10 = 2 × 5, but 2s are always more, so count 5s. **Eg:** Trailing zeros in $100!$ = 20 + 4 = **24**
Trailing zeros in a power a^b	Count of 10s in a^b	Prime factorize a, find min(power of 2, power of 5) in result
Trailing zeros in a number like 1000	Count number of zeros at the end	Eg: 1000 → 3 trailing zeros
Zeros in decimal	Depends on denominator of fraction	Eg: 1/10 = 0.1 (1 zero), 1/100 = 0.01 (2 zeros)
Ending zeros in product of numbers	Count net power of 10 in the product	Eg: $2^3 \times 5^4$ → Min(3,4) = 3 → **3 zeros**

Remainder Theorem:

Type of Division	Formula / Concept	Example
$f(a)$ is remainder when polynomial $f(x)$ is divided by $x - a$	Remainder = $f(a)$	Eg: Remainder when $f(x) = x^2 + 3x + 2$ divided by $x - 2$ is $f(2) = 4 + 6 + 2 = **12**$
$a^n \mod b$ when a and b are coprime	Use **Euler's Theorem:** $a^{\phi(b)} \equiv 1 \mod b$	Eg: $7^{100} \mod 10$: φ(10)=4 → $7^4 \equiv 1 \mod 10$ → $7^{100} \equiv 1^{25} = 1$
$a^n \mod b$ for small values	Use cyclicity / patterns	Eg: $3^n \mod 4$ → 3, 1, 3, 1... pattern of 2
Divisibility of large numbers	Break number using modulo	Eg: Find remainder when 111...111 (15 times) is divided by 9: Since sum of digits = 15 → remainder = **6**
Division of powers like $2^{20} \mod 7$	Use cyclicity of powers	Powers of 2 mod 7: 2, 4, 1 (repeat every 3) → 20 mod 3 = 2 → answer is $2^2 = **4**$
Special case: $a^n - b^n$ divisible by $a - b$	Always divisible	Eg: $7^5 - 3^5$ divisible by $7 - 3 = 4$

Divisibility Rules:

Divisor	Divisibility Rule	Example
2	If the number ends in 0, 2, 4, 6, or 8.	134 **is** divisible by 2 (ends in 4).
3	If the sum of the digits is divisible by 3.	123 → 1+2+3=6 → divisible by 3.
4	If the last **two digits** form a number divisible by 4.	316 **is** divisible by 4 (16 ÷ 4 = 4).
5	If the number ends in **0 or 5**.	475, 900 both end in 5 or 0.
6	If the number is divisible by **both 2 and 3**.	132 is even and 1+3+2=6 (div. by 3).
7	Double the last digit, subtract from the rest; if result is divisible by 7.	203: 20 – 6 = 14 → divisible by 7.
8	If the last **three digits** form a number divisible by 8.	1216 → 216 ÷ 8 = 27.
9	If the sum of the digits is divisible by 9.	729 → 7+2+9=18 → divisible by 9.
10	If the number ends in **0**.	370, 800 end in 0.
11	Alternating sum of digits is divisible by 11 (e.g. +, –, +, – ...).	121 → 1–2+1 = 0 → divisible by 11.
12	If the number is divisible by both **3 and 4**.	240: 2+4+0=6 (div. by 3), 40 div. by 4.
13	Multiply last digit by 9, subtract from rest. If result divisible by 13.	637: 63 – (7×9)=63–63=0 → OK.
14	If the number is divisible by **2 and 7**.	196 is even and divisible by 7.
15	If the number is divisible by **3 and 5**.	225 is div. by 3 (2+2+5=9) and ends 5.
16	If the last **4 digits** form a number divisible by 16.	65472 → 5472 ÷ 16 = 342.
17	Subtract 5 × last digit from the rest. Repeat if needed.	221 → 22 – 5×1 = 17 → divisible.
18	If the number is divisible by **2 and 9**.	198 → even, 1+9+8=18 → divisible.
19	Multiply last digit by 2, add to rest. If divisible by 19.	133 → 13 + (2×3) = 13+6 = 19 → OK.
20	If the number ends in **00, 20, 40, 60, or 80**.	560 ends in 60 → divisible by 20.