Quant Mega Quiz for SSC CGL Tier – 2 (Solutions)

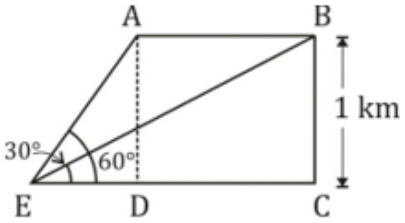
S1. Ans.(d)

Sol. Given: Height of aeroplane from the ground AD = 1 km; Initial angle of elevation = 60° and angle of elevation after 10 second = 30°.

Let A be the initial position of the aeroplane and E be the position of observer.

And B be the position of the aeroplane after 10 sec.

Therefore ∠AED = 60°, ∠BEC = 30° and AB = CD.



We know that in ΔAED, $\frac{AD}{DE} = \tan 60° = \sqrt{3}$

or $\frac{1}{DE} = \sqrt{3}$ or $DE = \frac{1}{\sqrt{3}}$

Similarly, in ΔBEC, $\frac{BC}{DE + CD} = \tan 30°$

or $\frac{1}{DE + CD} = \frac{1}{\sqrt{3}}$ or $DE + CD = \sqrt{3}$

or $CD = \sqrt{3} - DE = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$.

Therefore speed of the aeroplane per hour

$$= \frac{Distance\ AB}{Time\ taken\ to\ travel} = \frac{\frac{2}{\sqrt{3}}}{\frac{10}{60 \times 60}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{60 \times 60}{10} = 240\sqrt{3}\ km/h.$$

S2. Ans.(c)

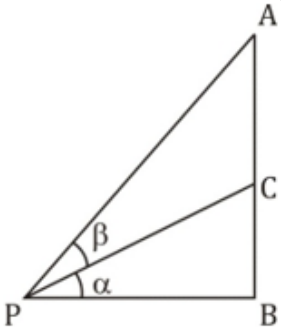
Sol.

Given: Vertical pole = AB; Middle point of AB = C or

AC = $\frac{AB}{2}$; Angle ∠APC = β and BP = n AB.

Let ∠CPB = α .

Therefore ∠APB = α + β.



We know that in ΔCPB, $\tan\alpha = \frac{CB}{BP} = \frac{CB}{nAB} = \frac{AB}{2nAB} = \frac{1}{2n}$.

Similarly, in ΔAPB, $\tan(\alpha + B) = \frac{AB}{BP} = \frac{AB}{nAB} = \frac{1}{n}$.

We also know that $\beta = \alpha + \beta - \alpha$ or $\tan\beta = \tan\{(\alpha + \beta) - \alpha\}$

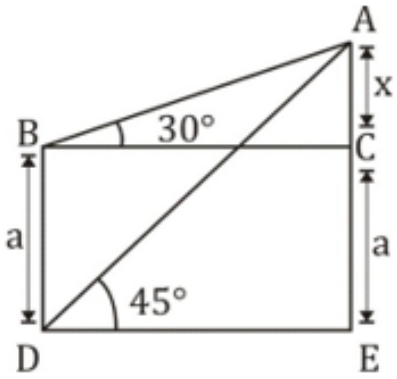
$$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\tan\alpha} = \frac{\frac{1}{n} - \frac{1}{2n}}{1 + \frac{1}{n} \times \frac{1}{2n}} = \frac{n}{2n^2 + 1}$$

$$\because \left\{\tan x - \tan y = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right\}$$

S3. Ans.(c)

Sol. Given: Height of building (BD) = a and angle of elevation of top of the tower A from the top B = 30° and angle of elevation of top of the tower from bottom D of the building = 45°.

Let x we the distance between the points A and C.



We know that in ΔABC, $\tan 30° = \frac{AC}{BC}$

or $\frac{1}{\sqrt{3}} = \frac{x}{BC}$ or $BC = x\sqrt{3}$.

And in ΔADE, $\tan 45° = \frac{a+x}{BC}$

or $1 = \frac{a+x}{x\sqrt{3}}$ or $x\sqrt{3} = a + x$

or $x(\sqrt{3} - 1) = a$ or $x = \frac{a}{\sqrt{3}-1}$

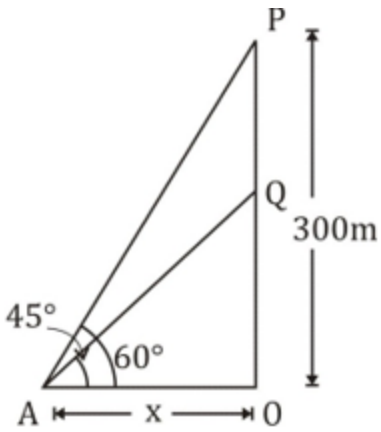
Therefore height of the tower.

$$= a + x = a + \frac{a}{\sqrt{3}-1} = a\left[\frac{\sqrt{3}-1+1}{\sqrt{3}-1}\right] = \left(\frac{a\sqrt{3}}{\sqrt{3}-1}\right) \times \left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{a(3+\sqrt{3})}{2}.$$

S4. Ans.(c)

Sol. Given: Height of an aeroplane from ground (OP) = 300 metres; Angle of elevation of the plane P= 60° and angle of elevation of the aeroplane Q.= 45°.

Let x be the height of the lower aeroplane OQ.



We know that in Δ PAO,

$\tan 60° = \frac{PO}{OA}$

or $\sqrt{3} = \frac{300}{x}$ or $x = \frac{300}{\sqrt{3}} = 100\sqrt{3}\ m.$

Since the ∠QAO is equal to 45°, therefore height of the lower aeroplane from the ground (x) = $100\sqrt{3}$ m.

S5. Ans.(b)

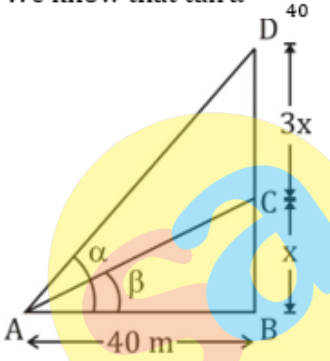
Sol.

Given: Height of upper portion DC of the pole = 3x (where 4x is the height of pole);

Angle subtended by it (α – β) = $\tan^{-1}\left(\frac{3}{5}\right)$; Distance AB = 40 m.

...(where α and β are the angles subtended by DB and CB).

We know that $\tan\alpha = \frac{4x}{40} = \frac{x}{10}$ and $\tan\beta = \frac{x}{40}$.



We also know that

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$or\ \ \tan\left(\tan^{-1}\frac{3}{5}\right) = \frac{\frac{x}{10} - \frac{x}{40}}{1 + \left(\frac{x}{10} \times \frac{x}{40}\right)}$

or $\frac{3}{5} = \frac{30x}{400 + x^2}$

or $x^2 - 50x + 400 = 0$

or $x^2 - 40x - 10x + 400 = 0$

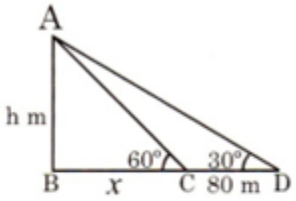
or $(x - 40)(x - 10) = 0.$

Therefore either x = 40 or 10.

Thus total height of the pole (4x) = 4 × 10 = 40 m.(from options)

S6. Ans.(b)
Sol.

Let AB = h m



And BC = x m

From ΔABC,

$\tan 60° = \frac{AB}{BC} = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}x$ .....(i)

$\tan 30° = \frac{h}{x+80}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+80}$

$\Rightarrow 3x = x + 80$

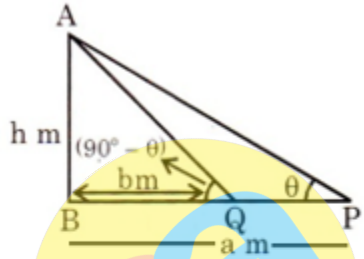
$\Rightarrow 2x = 80$

$\therefore x = 40$ m

Hence, side = $\frac{40}{\sqrt{2}} = 20\sqrt{2}m$

S7. Ans.(c)
Sol.

Let AB = h m



∠APB = θ°

From Δ ABP,

$\tan\theta = \frac{AB}{BP} = \frac{h}{a}$ .....(i)

From Δ ABQ,

$\tan(90° - \theta) = \frac{AB}{BQ}$

$\cot\theta = \frac{h}{b}$

$\tan\theta = \frac{b}{h}$ .....(ii)

From equation (i) and (ii),

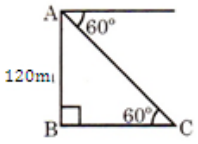
$\frac{h}{a} = \frac{b}{h}$

$h^2 = ab$

$\therefore h = \sqrt{ab}$ m

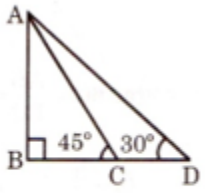
AQ=$\sqrt{ab + b^2} = \sqrt{b(a+b)}$

S8. Ans.(c)
Sol.
In right Δ ABC,



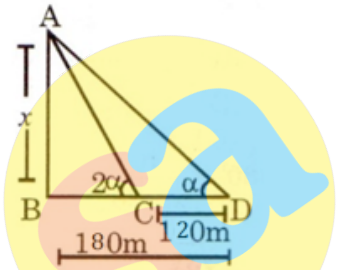
$\tan 60° = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{120}{BC}$
$\Rightarrow BC = \frac{120}{\sqrt{3}}$
$\Rightarrow BC = \frac{120\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$
$\therefore BC = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$ m

S9. Ans.(c)
Sol.
Required distance (d) = $h\ (\cot\theta_1 - \cot\theta_2)$



$= h(\cot\theta_1 - \cot\theta_2)$
$= 100(\cot 30° - \cot 45°)$
$= 100\left(\sqrt{3} - 1\right)$ m

S10. Ans.(a)
Sol.
In right Δ ABD,



$\tan\alpha = \frac{x}{180}$
$x = 180\tan\alpha$ .....(i)
In right Δ ABC,
$\tan 2\alpha = \frac{x}{60}$
$\tan 2\alpha = \frac{180\tan\alpha}{60}$
$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{180\tan\alpha}{60}$
$180(1 - \tan^2\alpha) = 120$
$180 - 120 = 180\tan^2\alpha$
$\frac{60}{180} = \tan^2\alpha \Rightarrow \tan^2\alpha = \frac{1}{3}$
$\Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$
$\therefore x = 180\tan\alpha = 180\times\frac{1}{\sqrt{3}} = 60\sqrt{3}$

S11. Ans.(b)

Sol.

$\cos(\theta - A) = a$

$\cos(\theta - B) = b$

Put $\theta = 90°$

$\cos(90° - A) = a$

$a = \sin A$

$b = \sin B$

Put $A = 60°$

$B = 30°$

$a = \frac{\sqrt{3}}{2}$

$b = \frac{1}{2}$

$\therefore \sin^2(A - B) + 2ab\cos(A - B)$

$\sin^2(60° - 30°) + 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}\cos(60° - 30°)$

$\Rightarrow \frac{1}{4} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{4} + \frac{3}{4} = 1$

Now check the option

Option (b):- $a^2 + b^2$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ (Satisfy)

S12. Ans.(b)

Sol.

$\frac{1}{2}\left(\cos 15°.\cos 7\frac{1}{2}°.\cos 82\frac{1}{2}°\right) \times 2$

$\Rightarrow \frac{1}{2}\left(\cos 15° . 2 \times \cos 7\frac{1}{2}°.\sin 7\frac{1}{2}°\right)$

$\Rightarrow \frac{1}{2}\cos 15° . \sin 15°$

Multiply and divide by 2

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 2\cos 15° \sin 15°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 2 \times 15°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 30°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

S13. Ans.(a)

Sol.

$k = (\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$ ...(i)

$k = (\sec\alpha - \tan\alpha)(\sec\beta - \tan\beta)(\sec\gamma - \tan\gamma)$ ...(ii)

Multiplying equation (i) & (ii)

$k^2 = (\sec^2\alpha - \tan^2\alpha)(\sec^2\beta - \tan^2\beta)(\sec^2\gamma - \tan^2\gamma)$

$k^2 = 1 \Rightarrow k = \pm 1$

S14. Ans.(b)

Sol.

$$\frac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{\cos 40^\circ - \cos 20^\circ + \cos 40^\circ}{\sin 20^\circ}$$

Using formula, $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

$$= \frac{2\sin 30^\circ . \sin(-10^\circ) + \cos 40^\circ}{\sin 20^\circ}$$

$$\left[\begin{matrix}\sin(-\theta) = -\sin\theta \\ \cos(90^\circ - 50^\circ) = \sin 50^\circ\end{matrix}\right]$$

$$= \frac{\sin 50^\circ - \sin 10^\circ}{\sin 20^\circ}$$

Use $\sin C - \sin D$

$$= 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$$

$$= \frac{2\sin 20^\circ . \cos 30^\circ}{\sin 20^\circ}$$

$= 2\cos 30^\circ = \sqrt{3}$

S15. Ans.(b)

Sol.

$$2\sqrt{2}\sin 10^\circ \times \left(\frac{1}{2\cos 5^\circ} + \frac{\cos 40^\circ}{\sin 5^\circ} - 2\sin 35^\circ\right)$$

$$\Rightarrow 2\sqrt{2}\sin 10^\circ \times \left(\frac{\sin 5^\circ + 2\cos 5^\circ\cos 40^\circ - 2\sin 35^\circ . 2\sin 5^\circ\cos 5^\circ}{2\sin 5^\circ\cos 5^\circ}\right)$$

Using formula, $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$\Rightarrow 2\sqrt{2}\sin 10^\circ \times \left(\frac{\sin 5^\circ + \cos 45^\circ + \cos 35^\circ - 2\sin 35^\circ\sin 10^\circ}{\sin 10^\circ}\right)$$

$\Rightarrow 2\sqrt{2}\,(\sin 5^\circ + \cos 45^\circ + \cos 35^\circ - 2\sin 35^\circ.\sin 10^\circ)$

$$\Rightarrow 2\sqrt{2}\left[\sin 5^\circ + \frac{1}{\sqrt{2}} + \cos 35^\circ - (\cos 25^\circ - \cos 45^\circ)\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5^\circ + \frac{1}{\sqrt{2}} + \cos 35^\circ - \cos 25^\circ + \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5^\circ + \cos 35^\circ - \cos 25^\circ + \frac{2}{\sqrt{2}}\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5^\circ - 2\sin 30^\circ . \sin 5^\circ + \frac{2}{\sqrt{2}}\right]$$

$\Rightarrow 2\sqrt{2}\,[\sin 5^\circ - \sin 5^\circ + \sqrt{2}] = 4$

S16. Ans.(b)

Sol.

Applying C & D

$$\frac{2\tan\theta}{2\cot\theta} = \frac{3}{1}$$

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{3}{1}$$

$$\sin^2\theta = 3(1 - \sin^2\theta)$$

$$4\sin^2\theta = 3$$

$$\sin\theta = \sqrt{3}/2$$

S17. Ans.(c)

Sol.

$$\Rightarrow 2\frac{\cos 22^\circ}{\cos 22^\circ} - \frac{2\tan 75^\circ}{5\tan 75^\circ} - \frac{3\tan 20^\circ . \cot 20^\circ \tan 40^\circ \cot 40^\circ}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5}$$

$$= 1$$

S18. Ans.(c)

Sol.

$$(\sin^2\theta + \cos^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$$

$$2\cos^2\theta - 1 = \frac{2}{3}$$

S19. Ans.(a)

Sol.

$$\frac{\sin\alpha}{\cos(30^\circ+\alpha)} = 1$$

$$\frac{\sin\alpha}{\sin(90 - 30 - \alpha)} = 1$$

$$\frac{\sin\alpha}{\sin(60 - \alpha)} = 1$$

$$\sin\alpha = \sin(60 - \alpha)$$

$$\alpha = 60 - \alpha$$

$$\alpha = 30^\circ$$

$$\sin 30^\circ + \cos 60^\circ$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

S20. Ans.(c)
Sol.

$$\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$$

$\sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$2\sin\theta = 4\cos\theta$

$\tan\theta = 2$

$\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \cos^2\theta(\tan^2\theta - 1)$

$$= \frac{\tan^2\theta - 1}{\sec^2\theta} = \frac{4-1}{1+4} = \frac{3}{5}$$

S21. Ans.(a)
Sol.

ATQ,

$9\cos\theta + 12\sin\theta = 15$

$$\Rightarrow \frac{9}{15}\cos\theta + \frac{12}{15}\sin\theta = 1$$

$\Rightarrow \cos^2\theta + \sin^2\theta = 1$

$$\cos\theta = \frac{9}{15}, \sin\theta = \frac{12}{15}$$

$$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{9}{12} = \frac{3}{4}$$

S22. Ans.(c)
Sol.

Put $\theta = 60°$

$\text{cosec}\theta - \cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \text{cosec}60° - \cot60° = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$ (Satisfied)

Now,

$$= \frac{\text{cosec}\theta}{\cot\theta}$$

$$= \frac{\text{cosec}60°}{\cot60°} = \frac{2\backslash\sqrt{3}}{1/\sqrt{3}} = 2$$

S23. Ans.(d)
Sol.

ATQ,

$9\sin^2\theta + 5\cos^2\theta = 8$

$\Rightarrow 9\sin^2\theta + 5(1 - \sin^2\theta) = 8$

$\Rightarrow 9\sin^2\theta + 5 - 5\sin^2\theta = 8$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$

$\therefore \cot 60° = \frac{1}{\sqrt{3}}$

S24. Ans.(d)
Sol.

ATQ,

$\tan\theta - \cot\theta = 0$

$\Rightarrow$ put $\theta = 45°$

$\tan 45° - \cot 45° = 0$

$1 - 1 = 0$

$0 = 0$ (satisfied)

$\therefore (\text{cosec}\theta - \sin\theta)(2\sec\theta - \cos\theta)(\cot\theta + \tan\theta)$

$= (\text{cosec}45° - \sin45°)(2\sec45° - \cos45°)(\cot45° + \tan45°)$

$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(2 \times \sqrt{2} - \frac{1}{\sqrt{2}}\right)(1 + 1)$

$= \frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{2}} \times 2 = 3$

S25. Ans.(c)
Sol.

ATQ,

$\sin A(1 + \sin A) = 1$

$\Rightarrow \sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

And,

$2\cos^2 A (1 + \cos^2 A)$

$= 2\cos^2 A + 2\cos^4 A$

$= 2(\cos^2 A + \sin^2 A) = 2$

S26. Ans.(b)

Sol.

$\cos(\theta - A) = a$

$\cos(\theta - B) = b$

Put $\theta = 90°$

$\cos(90° - A) = a$

$a = \sin A$

$b = \sin B$

Put $A = 60°$

$B = 30°$

$a = \frac{\sqrt{3}}{2}$

$b = \frac{1}{2}$

$\therefore \sin^2(A - B) + 2ab\cos(A - B)$

$\sin^2(60° - 30°) + 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}\cos(60° - 30°)$

$\Rightarrow \frac{1}{4} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{4} + \frac{3}{4} = 1$

Now check the option

Option (b):- $a^2 + b^2$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ (Satisfy)

S27. Ans.(b)

Sol.

$\frac{1}{2}\left(\cos 15°.\cos 7\frac{1}{2}°.\cos 82\frac{1}{2}°\right) \times 2$

$\Rightarrow \frac{1}{2}\left(\cos 15°.2 \times \cos 7\frac{1}{2}°.\sin 7\frac{1}{2}°\right)$

$\Rightarrow \frac{1}{2}\cos 15°.\sin 15°$

Multiply and divide by 2

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 2\cos 15° \sin 15°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 2 \times 15°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 30°$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

S28. Ans.(a)

Sol.

$k = (\sec\alpha + \tan\alpha\ (\sec\beta + \tan\beta)\ (\sec\gamma + \tan\gamma)$ ...(i)

$k = (\sec\alpha - \tan\alpha)\ (\sec\beta - \tan\beta)\ (\sec\gamma - \tan\gamma)$ ...(ii)

Multiplying equation (i) & (ii)

$k^2 = (\sec^2\alpha - \tan^2\alpha)(\sec^2\beta - \tan^2\beta)(\sec^2\gamma - \tan^2\gamma)$

$k^2 = 1 \Rightarrow k = \pm 1$

S29. Ans.(b)

Sol.

$$\frac{2\cos 40° - \cos 20°}{\sin 20°}$$

$$= \frac{\cos 40° - \cos 20° + \cos 40°}{\sin 20°}$$

Using formula, $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

$$= \frac{2\sin 30°.\sin(-10°) + \cos 40°}{\sin 20°}$$

$$\left[\begin{matrix}\sin(-\theta) = -\sin\theta \\ \cos(90° - 50°) = \sin 50°\end{matrix}\right]$$

$$= \frac{\sin 50° - \sin 10°}{\sin 20°}$$

Use $\sin C - \sin D$

$$= 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$$

$$= \frac{2\sin 20°.\cos 30°}{\sin 20°}$$

$= 2\cos 30° = \sqrt{3}$

S30. Ans.(b)

Sol.

$$2\sqrt{2}\sin 10° \times \left(\frac{1}{2\cos 5°} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right)$$

$$\Rightarrow 2\sqrt{2}\sin 10° \times \left(\frac{\sin 5° + 2\cos 5°\cos 40° - 2\sin 35°.2\sin 5°\cos 5°}{2\sin 5°\cos 5°}\right)$$

Using formula, $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$\Rightarrow 2\sqrt{2}\sin 10° \times \left(\frac{\sin 5° + \cos 45° + \cos 35° - 2\sin 35°\sin 10°}{\sin 10°}\right)$$

$\Rightarrow 2\sqrt{2}\ (\sin 5° + \cos 45° + \cos 35° - 2\sin 35°.\sin 10°)$

$$\Rightarrow 2\sqrt{2}\left[\sin 5° + \frac{1}{\sqrt{2}} + \cos 35° - (\cos 25° - \cos 45°)\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5° + \frac{1}{\sqrt{2}} + \cos 35° - \cos 25° + \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5° + \cos 35° - \cos 25° + \frac{2}{\sqrt{2}}\right]$$

$$\Rightarrow 2\sqrt{2}\left[\sin 5° - 2\sin 30°.\sin 5° + \frac{2}{\sqrt{2}}\right]$$

$\Rightarrow 2\sqrt{2}\ [\sin 5° - \sin 5° + \sqrt{2}] = 4$