Quant Mega Quiz for SSC CGL Tier – 2 (Solutions)

S1. Ans.(c)
Sol.
$\cos x \cos y + \sin x \sin y = -1$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$
$\cos(x - y) = -1$
$\cos(x - y) = \cos(180°)$
$x - y = 180°$
$\cos x + \cos y = 2\cos\frac{x+y}{2}.\cos\frac{x-y}{2}$
$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos 90°$
$\cos x + \cos y = 2\cos\frac{(x+y)}{2} \times 0$
$= 0$

S2. Ans.(b)
Sol.
$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta) + 1$
$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$
$= 2 - 3 + 1 = 0$

S3. Ans.(d)
Sol.
$\cos\theta = \frac{x^2 - y^2 \to \text{Base}}{x^2 + y^2 \to \text{Hypotenuse}}$
$\text{Perpendicular} = \sqrt{x^4 + y^4 + 2x^2y^2 - x^2 - y^2 + 2x^2y^2} = 2xy$
$\cot\theta = \frac{x^2 - y^2}{2xy}$

S4. Ans.(b)
Sol.
$x = \text{cosec}\,\theta - \sin\theta$
$y = \sec\theta - \cos\theta$
Put $\theta = 45°$
$x = 1/\sqrt{2}, y = 1/\sqrt{2}$
By options
$x^2y^2(x^2 + y^2 + 3)$
$= \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 \left[\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 3\right]$
$= 1$ satisfies

S5. Ans.(b)
Sol.
x tan 60° + cos 45° = sec 45°
$x(\sqrt{3}) + \frac{1}{\sqrt{2}} = \sqrt{2}$
$x\sqrt{6} + 1 = 2$
$x\sqrt{6} = 1$
$x = \frac{1}{\sqrt{6}}$
$x^2 + 1 = \frac{1}{6} + 1$
$= \frac{7}{6}$

S6. Ans.(c)
Sol.
sin (2x – 20°) = cos (2y + 20°)
sin [2x – 20°] = sin [90° – (2y + 20°)]
2x – 20° = 90° – (2y + 20°)
2x – 20° = 90° – 2y – 20°
2x + 2y = 90°
x + y = 45°
tan (x + y)
= tan 45°
= 1

S7. Ans.(b)
Sol.
$a^2 \sec^2 x - b^2 \tan^2 x = c^2$
$a^2 (1 + \tan^2 x) - b^2 \tan^2 x = c^2$
$a^2 + a^2 \tan^2 x - b^2 \tan^2 x = c^2$
$a^2 + \tan^2 x (a^2 - b^2) = c^2$
$a^2 - c^2 = \tan^2 x (b^2 - a^2)$
$\tan^2 x = \frac{a^2 - c^2}{b^2 - a^2}$
$\sec^2 x = 1 + \tan^2 x$
$= 1 + \frac{a^2 - c^2}{b^2 - a^2}$
$= \frac{b^2 - a^2 + a^2 - c^2}{b^2 - a^2}$
$= \frac{b^2 - c^2}{b^2 - a^2}$
$\sec^2 x + \tan^2 x$
$= \frac{a^2 - c^2}{b^2 - a^2} + \frac{b^2 - c^2}{b^2 - a^2}$
$= \frac{a^2 + b^2 - 2c^2}{b^2 - a^2}$

S8. Ans.(d)

Sol.

$$\frac{\sin\theta}{1+\cos\theta}+\frac{\sin\theta}{1-\cos\theta}$$
$$=\frac{\sin\theta-\sin\theta\cos\theta+\sin\theta+\sin\theta\cos\theta}{1-\cos^2\theta}$$
$$=\frac{2\sin\theta}{\sin^2\theta}$$
$= 2\ \text{cosec}\ \theta$

S9. Ans.(c)

Sol.

$\tan^4 + \tan^2\theta = 1$

$\tan^2\theta\,(1+\tan^2\theta) = 1$

$\tan^2\theta\sec^2\theta = 1$

$$\tan^2\theta = \frac{1}{\sec^2\theta}$$

$\tan^2\theta = \cos^2\theta$

$\cos^4 + \cos^2\theta$

$= \tan^4 + \tan^2\theta$

$= 1$

S10. Ans.(c)

Sol.

$8(\sin^6\theta + \cos^6\theta) - 12\,(\sin^4\theta + \cos^4\theta)$

$= 8\,(1 - 3\sin^2\theta\cos^2\theta) - 12(1 - 2\sin^2\theta\cos^2\theta)$

$= 8 - 24\sin^2\theta\cos^2\theta - 12 + 24\sin^2\theta\cos^2\theta$

$= -4$

S11. Ans.(b)

Sol.

If $\sin\alpha\sec\beta = 1$

then $\alpha + \beta = 90°$

$x + y + x - y = 90°$

$2x = 90°$

$x = 45°$

$\tan^2 45° + \cos^2 45° + \text{cosec}^2 45°$

$$= 1 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2}\right)^2$$
$$= 1 + \frac{1}{2} + 2$$
$$= \frac{2+1+4}{2} = \frac{7}{2}$$

S12. Ans.(b)

Sol.

$$\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}-\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

$$=\frac{\sin\theta\cot\theta-\sin\theta\operatorname{cosec}\theta-\sin\theta\cot\theta-\sin\theta\operatorname{cosec}\theta}{\cot^2\theta-\operatorname{cosec}^2\theta}$$

$$=\frac{-2\sin\theta\operatorname{cosec}\theta}{-(\operatorname{cosec}^2\theta-\cot^2\theta)}$$

$= 2$

S13. Ans.(a)

Sol.

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=K+\tan A+\cot A$$

$$\Rightarrow\frac{\tan^2 A}{\tan A-1}+\frac{\cot A}{1-\tan A}$$

$$\Rightarrow\frac{\cot A}{1-\tan A}-\frac{\tan^2 A}{1-\tan A}$$

$$=\frac{\cot A-\tan^2 A}{1-\tan A}$$

$$=\frac{1-\tan^3 A}{\tan(1-\tan A)}$$

$$=\frac{(1-\tan A)(1+\tan A+\tan^2 A)}{\tan A(1-\tan A)}$$

$$=\frac{1}{\tan A}+1+\tan A$$

$= 1+\tan A+\cot A$

$K = 1$

S14. Ans.(a)

Sol.

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\cos^3\theta}{\cos\theta-\sin\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^3\theta}{\sin\theta-\cos\theta}-\frac{\cos^3\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}$$

$$=\frac{(\sin\theta-\cos\theta)(\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta)}{\sin\theta-\cos\theta}$$

$= 1+\sin\theta\cos\theta$

$K = 1$

S15. Ans.(a)

Sol.

$\Rightarrow \frac{(1-\sin\theta+\cos\theta)^2}{(1+\cos\theta)(1-\sin\theta)}$

$= \frac{1+\sin^2\theta+\cos^2\theta-2\sin\theta+2\cos\theta-2\sin\theta\cos\theta}{(1+\cos\theta)(1-\sin\theta)}$

$= \frac{2-2\sin\theta+2\cos\theta-2\sin\theta\cos\theta}{1-\sin\theta+\cos\theta-\sin\theta\cos\theta}$

$= \frac{2(1-\sin\theta+\cos\theta-\sin\theta\cos\theta)}{(1-\sin\theta+\cos\theta-\sin\theta\cos\theta)}$

= 2

S16. Ans.(a)

Sol.

$(\sec^2\theta)^3 - (\tan^2\theta)^3 - 3\tan^2\theta\sec^2\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^4\theta + \tan^4\theta + \tan^2\theta\sec^2\theta) - 3\tan^2\theta\sec^2\theta$

$= \sec^4\theta + \tan^4\theta + \tan^2\theta\sec^2\theta - 3\tan^2\theta\sec^2\theta$

$= \sec^4\theta + \tan^4\theta - 2\tan^2\theta\sec^2\theta$

$= (\sec^2\theta - \tan^2\theta)^2$

$= (1)^2$

= 1

S17. Ans.(c)

Sol.

$\operatorname{cosec}^6\theta - \cot^6\theta - 3\cot^2\theta\operatorname{cosec}^2\theta$

$= (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^4\theta + \operatorname{cosec}^2\theta\cot^2\theta + \cot^4\theta) - 3\cot^2\theta\operatorname{cosec}^2\theta$

$= (\operatorname{cosec}^2\theta - \cot^2\theta)^2$

$= (1)^2 = 1$

S18. Ans.(b)

Sol.

$\frac{(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)}{(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2)}$

$= \frac{\left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}-2\right)}$

$= \frac{\left(\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{\cos\theta+\sin\theta}{\cos\theta\sin\theta}\right)\left(\frac{1-2\sin\theta\cos\theta}{\cos\theta\sin\theta}\right)}$

$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)(1-2\sin\theta\cos\theta)}$

$= \frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$

= 1

S19. Ans.(b)
Sol.

$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\tan\theta + 1 = \sqrt{2}$

$\tan\theta = \sqrt{2} - 1$

$\cot\theta = \frac{1}{\sqrt{2}-1}$

$= \sqrt{2} + 1$

S20. Ans.(b)
Sol.

$\cot\alpha = \frac{15 \to B}{8 \to P}$

$H = \sqrt{225 + 64} = 17$

$\sin\alpha = \frac{8}{17}$

$\cos\alpha = \frac{15}{17}$

$\frac{(2+2\sin\alpha)(1-\sin\alpha)}{(1+\cos\alpha)(2-2\cos\alpha)}$

$\Rightarrow \frac{\left(2+\frac{16}{17}\right)\left(1-\frac{8}{17}\right)}{\left(1+\frac{15}{17}\right)\left(2-\frac{30}{17}\right)}$

$= \frac{\frac{50}{17}\times\frac{9}{17}}{\frac{32}{17}\times\frac{4}{17}}$

$= \frac{225}{64}$

S21. Ans.(a)
Sol.

$(r\cos\theta - \sqrt{3})^2 + (r\sin\theta - 1)^2 = 0$

$\Rightarrow r\cos\theta - \sqrt{3} = 0 \; and \; r\sin\theta - 1 = 0$

$\Rightarrow r\cos\theta = \sqrt{3} \; and \; r\sin\theta = 1$

$\therefore r^2\cos^2\theta + r^2\sin^2\theta = 3 + 1$

$\Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow r^2 = 4 \Rightarrow r = 2$, -2

$\therefore \tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$

And r $\cos\theta = \sqrt{3} \Rightarrow \cos\theta = \frac{\sqrt{3}}{r}$

$\Rightarrow \sec\theta = \frac{r}{\sqrt{3}}$

$\therefore \frac{r\tan\theta + \sec\theta}{r\sec\theta + \tan\theta} = \frac{\frac{r}{\sqrt{3}} + \frac{r}{\sqrt{3}}}{\frac{r^2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$

$= \frac{r\left(\frac{2}{\sqrt{3}}\right)}{\frac{r^2+1}{\sqrt{3}}}$ taking positive value of r

$= \frac{2r}{r^2+1} = \frac{2\times2}{4+1} = \frac{4}{5}$

S22. Ans.(c)

Sol.

$x = a\ (\sin\theta + \cos\theta)\ and\ y = b\ (\sin\theta - \cos\theta)$

$\Rightarrow \frac{x}{a} = \sin\theta + \cos\theta\ and\ \frac{y}{b} = \sin\theta - \cos\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta$

$= 2(\sin^2\theta + \cos^2\theta) = 2$

S23. Ans.(a)

Sol.

$\sin 21° = \frac{x}{y}$

$\text{Cos } 21° = \sqrt{1 - \sin^2 21°}$

$= \sqrt{1 - \frac{x^2}{y^2}} = \frac{\sqrt{y^2 - x^2}}{y}$

$\therefore \sec 21° = \frac{y}{\sqrt{y^2 - x^2}}$

$\therefore \sec 21° - \sin 69°$

$= \sec 21° - \sin (90° - 21°)$

$= \sec 21° - \cos 21°$

$= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y}$

$= \frac{y^2 - (y^2 - x^2)}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$

S24. Ans.(b)

Sol.

$a\cos\theta + b\sin\theta = p$

$a\sin\theta - b\cos\theta = q$

On squaring and adding,

$a^2\cos^2\theta + b^2\sin^2\theta + 2\,a\,b$

$\sin\theta.\cos\theta + a^2\sin^2\theta + b^2$

$\cos^2\theta - 2\,a\,b\sin\theta.\cos\theta$

$= p^2 + q^2$

$\Rightarrow a^2\cos^2\theta + a^2\sin^2\theta + b^2$

$\sin^2\theta + b^2\cos^2\theta = p^2 + q^2$

$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = p^2 + q^2$

$\Rightarrow a^2 + b^2 = P^2 + q^2$

S25. Ans.(a)

Sol.

$\frac{5}{\sec^2\theta} + \frac{2}{1 + \cot^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + \frac{2}{\text{cosec}^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$

$= 5\,(\cos^2\theta + \sin^2\theta) = 5$

S26. Ans.(c)

Sol.

x = a sec$\alpha . cos\beta$

$\Rightarrow \frac{x}{\alpha} = sec\alpha . cos\beta$

Similarly,

$\frac{y}{b} = sec\alpha . sin\beta, \frac{z}{c} = tan\alpha$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \sec^2 \alpha . \cos^2 \beta + \sec^2 \alpha . \sin^2 \beta - \tan^2 \alpha$

$= \sec^2 \alpha(\cos^2 \beta + \sin^2 \beta) - \tan^2 \alpha$

$= \sec^2 \alpha - \tan^2 \alpha = 1$

S27. Ans.(d)

Sol.

$\tan^2 \theta = 1 - e^2$

$\therefore sec\theta + \tan^3 \theta . cosec\theta$

$= sec\theta + \tan^2 \theta . tan\theta . cosec\theta$

$= sec\theta + \tan^2 \theta . \frac{\sin\theta}{cos\theta} . \frac{1}{\sin\theta}$

$= \sec\theta . (1 + \tan^2 \theta)$

$= (1 + \tan^2 \theta)^{\frac{1}{2}} . (1 + \tan^2 \theta)$

$= (1 + \tan^2 \theta)^{\frac{3}{2}} = (1 + 1 - e^2)^{\frac{3}{2}}$

$= (2 - e^2)^{\frac{3}{2}}$

S28. Ans.(d)

Sol.

$\sin\theta = \cos(90° - \theta)$;

$\sin(90° - \theta) = \cos\theta$

$\therefore \sin 85° = \sin(90° - 5°) = \cos 5°$

$\therefore (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \cdots to\ 8\ terms + \sin^2 45° + \sin^2 90°$

$= 8 \times 1 + \frac{1}{2} + 1 = 9\frac{1}{2}$

S29. Ans.(b)

Sol.

$\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$

$= \frac{\cot 33° + \tan 53°}{\tan 33° + \cot 53°}$

$[\because \tan(90° - \theta) = cot\theta,$

$\cot(90° - \theta) = tan\theta]$

$= \frac{\frac{1}{\tan 33°} + \tan 53°}{\tan 33° + \frac{1}{\tan 53°}}$

$= \frac{1 + \tan 53° . \tan 33°}{\tan 33° . \tan 53° + 1} \times \frac{\tan 53°}{\tan 33°}$

$= \tan 53° . \cot 33°$

$= \cot 37° . \tan 57°$

S30. Ans.(a)

Sol.

$\cos\theta + \sec\theta = \sqrt{3}$

On cubing,

$(\cos\theta + \sec\theta)^3 = \left(\sqrt{3}\right)^3 = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\cos\theta.\sec\theta\,(\cos\theta + \sec\theta) = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta = 0$