Quant Mega Quiz for SSC CGL Tier – 2 (Solutions)

S1. Ans.(b)
Sol.

$(ab + bc + ca)^2$
$= a^2b^2 + b^2c^2 + c^2a^2 + 2abbc + 2abca + 2bcca$
Or, $0 = a^2b^2 + b^2c^2 + c^2a^2 + 2abc(b + a + c)$
$0 = a^2b^2 + b^2c^2 + c^2a^2 + 2pq$
$= a^2b^2 + b^2c^2 + c^2a^2 = -2pq$

S2. Ans.(a)
Sol.

$x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$
Here $x = y = 1$, satisfies the above equation
Then $x^2 + y^2 = 1^2 + 1^2 = 2$

S3. Ans.(c)
Sol.

$x^3 + y^3 = 35$ *and* $x + y = 5$
$\Rightarrow (x + y)(x^2 + y^2 - xy) = 35$
$x^2 + y^2 - xy = 7$ ....(i)
And $(x + y)^2 = 25$
$x^2 + y^2 + 2xy = 25$ ....(ii)
On solving equation (i) and (ii)
$3xy = 18$
$xy = 6$ ...(iii)
$\Rightarrow x + y = 5$ ...(iv)
from (iii) and (iv)

$$\frac{x + y}{xy} = \frac{5}{6}$$
$$\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$$

S4. Ans.(b)
Sol.

$x^4 + \frac{1}{x^4} = 23$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 5$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 5 - 2 = 3$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 3$

S5. Ans.(d)

Sol.

$\because a + b + c = 8$

$\Rightarrow (a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$

$= 0$

S6. Ans.(c)

Sol.

$27p^3 - \frac{1}{216} - \frac{a}{2}p^2 + \frac{1}{4}p$

$\Rightarrow \left(3p - \frac{1}{6}\right)^3 = \left(3 \times \frac{5}{18} - \frac{1}{6}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

S7. Ans.(b)

Sol.

$5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$

By hit & trial

$x = 4$

S8. Ans.(a)

Sol.

$x^2 + 2 = 2x$

$\Rightarrow x^2 - 2x + 2 = 0$

$\& \; x^4 - x^3 + x^2 + 2 = (x^2 - 2x + 2)(x^2 + x - 1) = 0$

S9. Ans.(b)

Sol.

$a : b = 2 : 3 \; \& \; b : c = 4 : 5$

$\Rightarrow a : b : c = 8 : 12 : 15$

$\Rightarrow a^2 : b^2 : bc = 64 : 144 : 180$

$\Rightarrow 16 : 36 : 45$

S10. Ans.(d)

Sol.

$\Rightarrow \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)\left(x^2 + \frac{1}{x^2} + 1\right)$

$\Rightarrow \left(x^2 - \frac{1}{x^2}\right)\left[\left(x^2 + \frac{1}{x^2}\right)^2 - 1\right]$

$\Rightarrow x^6 - \frac{1}{x^6}$

S11. Ans.(c)
Sol. Let the each ratio be K

$$\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)} = K$$

$x = K\,(b-c)(b+c-2a) = K(b^2-c^2) - K\,2a(b-c)$

$y = K\,(c-a)(c+a-2b) = K\,(c^2-a^2) - K\,2b\,(c-a)$

$z = K\,(a-b)(a+b-2c) = K\,(a^2-b^2) - K\,2c\,(a-b)$

On adding, $x+y+z=0$

S12. Ans.(c)
Sol.

$\because 10^{0.48} = x \;\&\; 10^{0.70} = y$

$\Rightarrow x^z = y^2$

$\Rightarrow (10^{0.48})^z = (10^{0.70})^2$

$\Rightarrow 10^{0.48z} = (10^{0.70})^2$

$= 0.48z = 1.4$

$= z = \frac{140}{48} = 2.9$

S13. Ans.(c)
Sol.

$x^{\frac{1}{3}} + y^{\frac{1}{3}} - z^{\frac{1}{3}} = 0$

then, $x+y-z = 3xy(-z)$ since, if, a+b+c=0 then $a^3+b^3+c^3 = 3abc$

taking cube both sides

$\Rightarrow (x+y-z)^3 = -27xyz$

$(x+y-z)^3 + 27xyz = 0$

S14. Ans.(b)
Sol.

$\because 2p + \frac{1}{p} = 4$

$\Rightarrow p + \frac{1}{2p} = 2$

Squaring both the sides

$p^2 + \frac{1}{4p^2} + 2 \times p \times \frac{1}{2p} = 4$

$\Rightarrow p^2 + \frac{1}{4p^2} = 3$

S15. Ans.(c)
Sol.

$\because x = 5 + 2\sqrt{6}$

$\Rightarrow \sqrt{x} = \sqrt{3} + \sqrt{2}$

$\& \quad \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2}$

$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$

S16. Ans.(a)

Sol.

$\because a+b+c=0$

$b+c=-a$ squaring both sides

$b^2+c^2=a^2-2bc$

$\Rightarrow \frac{a^2+b^2+c^2}{a^2-bc}-2=\frac{2(a^2-bc)}{(a^2-bc)}-2$

$=2-2=0$

S17. Ans.(c)

Sol.

$\Rightarrow x^4-2x^2y^2+y^2$

$\Rightarrow (x^2-y^2)^2 \Rightarrow \left[\left(p+\frac{1}{p}\right)^2-\left(p-\frac{1}{p}\right)^2\right]^2$

$\Rightarrow 4p^2\times\frac{4}{p^2}=4\times4=16$

S18. Ans.(c)

Sol.

$\frac{1}{x^{99}}+\frac{1}{x^{98}}+\frac{1}{x^{97}}+\frac{1}{x^{96}}+\frac{1}{x^{95}}+\frac{1}{x^{94}}+\frac{1}{x}-1$

Put $x=-1$

$=\frac{1}{(-1)^{99}}+\frac{1}{(-1)^{98}}+\frac{1}{(-1)^{97}}+\frac{1}{(-1)^{96}}+\frac{1}{(-1)^{95}}+\frac{1}{(-1)^{94}}+\frac{1}{(-1)}-1$

$=-1+1-1+1-1+1-1-1=-2$

S19. Ans.(b)

Sol.

$x^2=y+z$

$\Rightarrow x^2+x=x+y+z$

$\Rightarrow x(x+1)=x+y+z$ .... (i)

Similarly,

$y(y+1)=x+y+z$ ...(ii)

and

$z(z+1)=x+y+z$ ...(iii)

On adding (i), (ii)and (iii)

$\therefore \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

$=\frac{x}{x+y+z}+\frac{y}{x+y+z}+\frac{z}{x+y+z}$

$=\frac{x+y+z}{x+y+z}=1$

S20. Ans.(d)

Sol.

$(ad - bc)^2 + (ac + bd)^2$

$= a^2d^2 + b^2c^2 - 2acbd + a^2c^2 + b^2d^2 + 2abcd$

$= a^2d^2 + b^2c^2 + a^2c^2 + b^2d^2$

$= d^2(a^2 + b^2) + c^2(a^2 + b^2)$

$= (c^2 + d^2)(a^2 + b^2)$

$= 1 \times 2 = 2$

S21. Ans.(a)

Sol.

$1.5a = 0.04b$

$\frac{b}{a} = \frac{1.5}{0.04} = \frac{150}{4}$

By componendo and dividend

$\frac{b - a}{b + a} = \frac{150 - 4}{150 + 4} = \frac{146}{154} = \frac{73}{77}$

S22. Ans.(a)

Sol.

$\frac{a^2+b^2+ab}{a^3-b^3} = \frac{a^2+b^2+ab}{(a-b)(a^2+b^2+ab)}$

$= \frac{1}{a - b}$

$= \frac{1}{11 - 9} = \frac{1}{2}$

S23. Ans.(a)

Sol.

$p = 999$ (Given)

Now, $\sqrt[3]{p(p^2 + 3p + 3) + 1}$

$= \sqrt[3]{p^3 + 3p^2 + 3p + 1}$

$= \sqrt[3]{(p + 1)^3}$

$= p + 1 = 999 + 1$

$= 1000$

S24. Ans.(c)

Sol. Here x = 3, y = 5, z = 4

$= \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16}$

$= \frac{9}{9} + \frac{25}{25} + \frac{16}{16}$

$= 1 + 1 + 1 = 3$

S25. Ans.(b)
Sol.

Given $\frac{a^2-b^2-2bc-c^2}{a^2+b^2+2ab-c^2} = \frac{a^2-(b+c)^2}{(a+b)^2-c^2}$

$= \frac{[a+(b+c)][a-(b+c)]}{(a+b+c)(a+b-c)} = \frac{a-b-c}{a+b-c}$

S26. Ans.(d)
Sol. For Maximum value

$a = b = c = d = \frac{1}{4}$

$(1+a)(1+b)(1+c)(1+d)$

$= \left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)$

$= \left(\frac{5}{4}\right)^4$

S27. Ans.(a)
Sol.

$\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$\Rightarrow \frac{1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1} = a(2)^{\frac{2}{3}} + b(2)^{\frac{1}{3}} + c$

On multiplying numerator and denominator by $\left(2^{\frac{1}{3}}-1\right)$

$\Rightarrow \frac{2^{\frac{1}{3}}-1}{\left(2^{\frac{1}{3}}-1\right)\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}+1\right)} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\frac{2^{\frac{1}{3}}-1}{2-1} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\therefore a = 0, b = 1, c = -1$

$a + b + c = 0 + 1 - 1 = 0$

S28. Ans.(b)
Sol.

$a = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{(\sqrt{5}+1)^2}{5-1} = \frac{5+1+2\sqrt{5}}{4}$

$a = \frac{3+\sqrt{5}}{2}$

Similarly,

$b = \frac{3-\sqrt{5}}{2}$

$a + b = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2} = 3$

and $ab = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{(a+b)^2-ab}{(a+b)^2-3ab}$

$= \frac{3^2-1}{3^2-3} = \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$

S29. Ans.(c)

Sol. $x = \left(\sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}\right)$

On cubing both sides.

$$x^3 = \left(\sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3$$

$$= \left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)^3 + \left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3 + 3\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)$$

$$\left[\sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}\right]$$

$$= a+\sqrt{a^2+b^3} + a-\sqrt{a^2+b^3} + 3\left[\left(a+\sqrt{a^2+b^3}\right)\left(a-\sqrt{a^2+b^3}\right)\right]^{\frac{1}{3}} x$$

$$= 2a + 3\,(a^2 - a^2 - b^3)^{\frac{1}{3}}\, x$$

$$\therefore x^3 = 2a + 3\,(-b)x$$

$$x^3 + 3bx = 2a$$

S30. Ans.(c)

Sol. Average of x and $\frac{1}{x}$ = M

$$\Rightarrow \frac{x+\frac{1}{x}}{2} = \mathrm{M}, x + \frac{1}{x} = 2\mathrm{M}$$

On squaring both sides

$$\left(x+\frac{1}{x}\right)^2 = (2\mathrm{M})^2$$

$$x^2 + \frac{1}{x^2} + 2 = 4\mathrm{M}^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 4\mathrm{M}^2 - 2$$

Average $x^2 + \frac{1}{x^2} = \frac{x^2+\frac{1}{x^2}}{2}$

$$= \frac{4\mathrm{M}^2-2}{2} = \frac{2(2\mathrm{M}^2-1)}{2}, = 2\mathrm{M}^2 - 1$$