Quant Mega Quiz for SSC CHSL (Solutions)

S1. Ans.(b)
Sol.
$x^2 - 3x + 1 = 0$
$x^2 + 1 = 3x$
$x\left(x + \frac{1}{x}\right) = 3x$
$x + \frac{1}{x} = 3$
Cubing both sides.
$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$
$x^3 + \frac{1}{x^3} + 9 = 27$
$x^3 + \frac{1}{x^3} = 27 - 9$
$= 18$

S2. Ans.(a)
Sol.
$x + \frac{1}{4x} = \frac{3}{2}$
$2x + \frac{1}{2x} = 3$
Cubing both sides
$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}\left(2x + \frac{1}{2x}\right) = 27$
$8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$
$8x^3 + \frac{1}{8x^3} = 18$

S3. Ans.(a)
Sol.
$\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$
$\frac{1}{x+y} = \frac{x+y}{xy}$
$xy = (x+y)^2$
$x^2 + y^2 + 2xy = xy$
$x^2 + y^2 + xy = 0$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
$= (x - y) \times 0$
$= 0$

S4. Ans.(c)
Sol.
x = a (b – c)
y = b (c – a)
z = c (a – b)
$\frac{x}{a}$ = b – c = A
$\frac{y}{b}$ = c – a = B
$\frac{z}{c}$ = a – b = C
A + B + C = b – c + c – a + a – b
A + B + C = 0
If A + B + C = 0
Then
$A^3 + B^3 + C^3 - 3ABC = 0$
$A^3 + B^3 + C^3 = 3ABC$
$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$
$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c}$
$= 3\frac{xyz}{abc}$

S5. Ans.(c)
Sol.
xy (x + y) = 1
$x + y = \frac{1}{xy}$
cubing both sides
$\Rightarrow (x + y)^3 = \frac{1}{x^3y^3}$
$\Rightarrow x^3 + y^3 + 3xy(x + y) = \frac{1}{x^3y^3}$
$x + y = \frac{1}{xy}$
$x^3 + y^3 + 3xy \times \frac{1}{xy} = \frac{1}{x^3y^3}$
$\frac{1}{x^3y^3} - x^3 - y^3 = 3$

S6. Ans.(d)
Sol.
$x^4 + \frac{1}{x^4} = 119$
Adding 2 both sides
$x^4 + \frac{1}{x^4} + 2 = 119 + 2$
$\left(x^2 + \frac{1}{x^2}\right)^2 = 121$
$x^2 + \frac{1}{x^2} = 11$

Adding 2 both sides

$x^2 + \frac{1}{x^2} + 2 = 11 + 2$

$\left(x + \frac{1}{x}\right)^2 = 13$

$x + \frac{1}{x} = \sqrt{13}$

Cubing both sides

$\left(x + \frac{1}{x}\right)^3 = (\sqrt{13})^3$

$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 13\sqrt{13}$

$x^3 + \frac{1}{x^3} + 3\sqrt{13} = 13\sqrt{13}$

$x^3 + \frac{1}{x^3} = 10\sqrt{13}$

S7. Ans.(b)

Sol.

$3x + \frac{1}{2x} = 5$

Multiplying both side by 2/3

$\frac{2}{3} \times 3x + \frac{2}{3} \times \frac{1}{2x} = 5 \times \frac{2}{3}$

$2x + \frac{1}{3x} = \frac{10}{3}$

Cubing both sides

$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x}\left(2x + \frac{1}{3x}\right) = \frac{1000}{27}$

$8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$

$8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000-180}{27} = \frac{820}{27} = 30\,\frac{10}{27}$

S8. Ans.(a)

Sol.

$x + y = z$

$x + y - z = 0$

$x + y + (-z) = 0$

if $A+B+C = 0$

then

$a^3 + b^3 + c^3 - 3abc = 0$

$a = x, \quad b = y \quad c = -z$

$x^3 + y^3 - z^3 + 3xyz = 0$

S9. Ans.(c)

Sol.

$\frac{a}{b} + \frac{b}{a} = 1$

$a^2 + b^2 = ab$

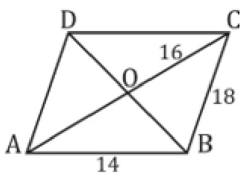
$a^2 + b^2 - ab = 0$

$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$= (a+b) \times 0$

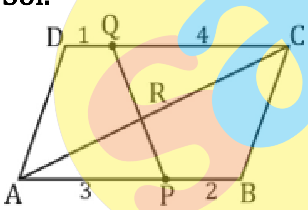
$= 0$

S10. Ans.(c)
Sol.
$x = 2 - 2^{1/3} + 2^{2/3}$
$x - 2 = 2^{2/3} - 2^{1/3}$ ...(i)
Cubing both sides
$x^3 - 8 - 3x \times 2(x-2) = (2^{2/3})^3 - (2^{1/3})^3 - 3\,(2)^{2/3}\,(2)^{1/3}\,(2^{2/3} - 2^{1/3})$ ....(ii)
From (i) & (ii)
$x^3 - 8 - 6x^2 + 12x = 4 - 2 - 6\,(x - 2)$
$x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$
$x^3 - 6x^2 + 18x = 22$
Adding 18 both sides
$x^3 - 6x^2 + 18x + 18 = 22 + 18$
$= 40$

S11. Ans.(d)
Sol.


$2(AB^2 + BC^2) = AC^2 + BD^2$
$\Rightarrow BD^2 = [2\,(196 + 324)] - 256$
$BD^2 = 784 \Rightarrow BD = 28$ cm

S12. Ans.(d)
Sol.
$$ar(\Delta OAB) = \frac{1}{4} ar(\| \text{gm } ABCD)$$
$$= \frac{1}{4} \times 56 = 14 \text{ cm}^2$$

S13. Ans.(b)
Sol.


AB ∥ CD
In ΔARP and ΔQRC
$\Rightarrow \angle RAP = \angle RCQ$ (alternate interior angles)
$\Rightarrow \angle RPA = \angle RQC$
$\Delta ARP \sim \Delta RCQ$ (similar triangle)
$$\Rightarrow \frac{RC}{AR} = \frac{QC}{AP} = \frac{4}{3}$$
Adding 1 both the sides
$$\Rightarrow \frac{RC}{AR} + 1 = \frac{4}{3} + 1$$
$$\Rightarrow \frac{RC + AR}{AR} = \frac{7}{3}$$
$$\Rightarrow AR = \frac{3}{7} AC$$

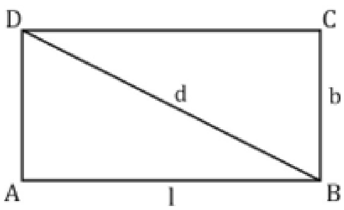
S14. Ans.(b)

Sol.

EF = AD = 8 ($\because$ EADF is a rectangle)
CD = 22 – 16 = 6
So, In right angled ΔADC,
AC = $\sqrt{8^2 + 6^2}$ = 10
$\therefore$ length of the line joining the mid-points of AB & BC
= $\frac{1}{2}$(AC) = 5

S15. Ans.(a)

Sol.



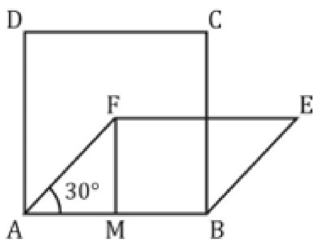
Perimeter (p) = 2 (l + b)
Diagonal (d) = $\sqrt{l^2 + b^2}$
$\Rightarrow d^2 = l^2 + b^2$
$\Rightarrow \frac{P^2}{4} = l^2 + b^2 + 2lb$
From equation [(ii) – (i)] we get
$\frac{P^2}{4} - d^2 = 2lb$
Formula
$(l^2 - b^2) = l^2 + b^2 - 2lb$
Now, putting value of P and d
$\because (l-b)^2 = d^2 - \frac{P^2}{4} + d^2 = \frac{8d^2 - P^2}{4}$
$\Rightarrow l - b = \sqrt{\frac{8d^2 - P^2}{4}}$ unit

S16. Ans.(c)

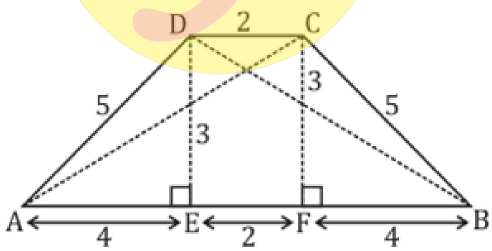
Sol.

P = 2 (l + b)
$\therefore$ b, l and P are in G.P
$\Rightarrow l^2 = bp$
$\Rightarrow l^2 = b \times 2(l + b)$
$\Rightarrow \frac{l^2}{b^2} = 2\left(\frac{1}{b} + 1\right)$
$\Rightarrow \left(\frac{l}{b}\right)^2 - 2\left(\frac{1}{b}\right) - 2 = 0$
$\frac{l}{b} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 + 2\sqrt{3}}{2}$
$= (\sqrt{3} + 1) : 1$
(Negative factor can not consider other wise value of l/b will be negative which is not possible)

S17. Ans.(a)
Sol.
Perimeter = 4a = 150 ⇒ 2a = 75 cm.
$4a^2 = {d_1}^2 + {d_2}^2 \Rightarrow (2a)^2 = (d_1)^2 + (d_2)^2$
$\Rightarrow (75)^2 = (50)^2 + {d_2}^2$
$\Rightarrow (75)^2 - (50)^2 = {d_2}^2$
$\Rightarrow {d_2}^2 = (75 + 50)(75 - 50)$
$= 125 \times 25 = 25 \times 25 \times 5$
$d^2 = 25\sqrt{5}$
$\Rightarrow \text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 50 \times 25\sqrt{5} + +$
$= 625\sqrt{5}\ \text{cm}^2$

S18. Ans.(b)
Sol.


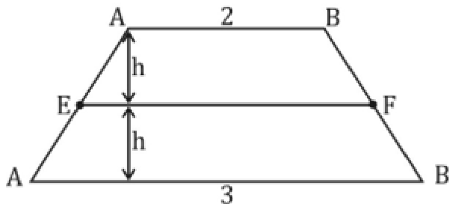
ABCD is a square and ABEF is a rhombus
$\sin 30^\circ = \frac{FM}{AF} = \frac{1}{2}$
$\Rightarrow FM = \frac{AF}{2}, AF = AB = a$
Area of square = a^2 (AB = AD = a)
Area of rhombus = AB × FM
$= a \times \left(\frac{a}{2}\right) = \frac{a^2}{2}$
$\therefore \frac{\text{Area of square}}{\text{Area of rhombus}} = \left(\frac{a^2}{a^2/2}\right)$
$= \frac{2}{1}$

S19. Ans.(b)
Sol.


In ΔACF
$AC = \sqrt{6^2 + 3^2} = \sqrt{45}$
$= \sqrt{9 \times 5} = 3\sqrt{5}$
In Δ BDE,
$DB = \sqrt{6^2 + 3^2} = 3\sqrt{5}$
Sum of diagonals
$= 3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5}$ cm

S20. Ans.(a)

Sol.



By property of trapezium

$$EF = \frac{1}{2}(AB + CD) = \frac{1}{2}(2 + 3) = \frac{5}{2}$$

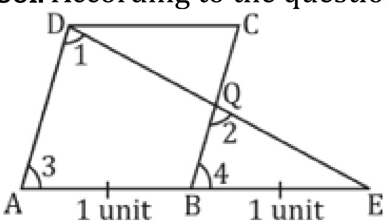
$$\frac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} = \frac{\frac{1}{2}\left(2 + \frac{5}{2}\right) \times h}{\frac{1}{2}\left(3 + \frac{5}{2}\right) \times h} = \frac{9}{11}$$

Alternate

$$\frac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} = \frac{3a + b}{a + 3b} = \frac{9}{11}$$

S21. Ans.(b)

Sol. According to the question



AD ∥ BC and AB ∥ DC

Point B is the midpoint of AE

$\angle 1 = \angle 2$ (Alternate Angle)

$\angle 3 = \angle 4$ (Alternate Angle)

$\therefore \Delta EQB \sim \Delta EDA$

$$\therefore \frac{EB}{EA} = \frac{EQ}{ED} = \frac{QB}{AD}$$

$$\Rightarrow \frac{1}{2} = \frac{QB}{AD}$$

$$\Rightarrow \frac{QB}{AD} = \frac{1}{2}$$

If AD = 2

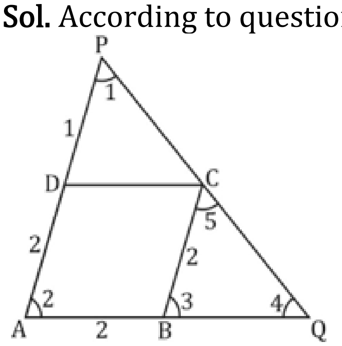
QB = 1

Then QC = 1

$\therefore$ Q divides BC in the ratio (1 : 1)

S22. Ans.(a)

Sol. According to questions. Given:



ABCD is a rhombus

AB = BC = CD = DA

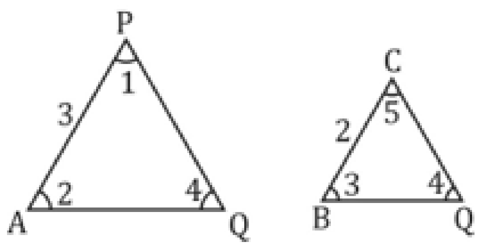
$\Rightarrow DP = \frac{1}{2} AB$

$\Rightarrow \frac{DP}{AB} = \frac{1}{2}$

In a rhombus $\angle 2 = \angle 3$

$\therefore \Delta APQ \sim \Delta BCQ$

($\because \angle Q$ is common and $\angle 2 = \angle 3$)



$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BQ}$ $\quad \frac{AQ}{BQ} = \frac{3}{2}$

$\Rightarrow \frac{AB+BQ}{BQ} = \frac{3}{2}$ $\quad (\therefore AQ = AB + BQ)$

$\Rightarrow \frac{AB}{BQ} + 1 = \frac{3}{2}$

$\Rightarrow \frac{AB}{BQ} = \frac{3}{2} - 1$

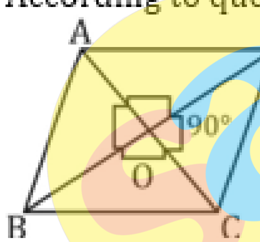
$\Rightarrow \frac{AB}{BQ} = \frac{1}{2}$

$\Rightarrow \therefore \frac{BQ}{AB} = \frac{2}{1}$

S23. Ans.(b)

Sol.

According to question,



$OB^2 + OC^2 = BC^2$ (i)

$OB^2 + OA^2 = AB^2$ (ii)

$OA^2 + OD^2 = AD^2$ (iii)

[By Pythagoras theorem]

$OC^2 + OD^2 = CD^2$

Add equation (i), (ii), (iii) and (iv)

$\Rightarrow 2(OB^2 + OC^2 + OD^2 + OA^2)$

$= BC^2 + AB^2 + AD^2 + CD^2$

$\Rightarrow 2BC^2 + 2AD^2$

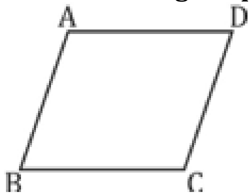
$= BC^2 + AB^2 + AD^2 + CD^2 + BC^2 + AD^2$

$= AB^2 + CD^2$

Or $AB^2 + CD^2 = BC^2 + DA^2$

S24. Ans.(c)

Sol. According to questions.



Given:

Ratio of ∠A and ∠B is 4 : 5

$\Rightarrow \frac{\angle A}{\angle B} = \frac{4}{5}$

We know that ∠A + ∠B = 180°

9 units = 180°

1 units = 20°

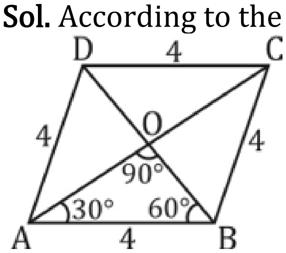
∠A = 4 units = 4 × 20° = 80°

∠A = ∠C = 80°

[Opposite ∠ of rhombus are equal.]

S25. Ans.(d)

Sol. According to the question



Given: ∠B = 120°

In a rhombus diagonal are angle bisector and diagonal cut at right triangle.

$\therefore \sin 30° = \frac{P}{H} = \frac{BO}{AB} = \frac{1}{2} = \frac{BO}{4}$

BO = 2 cm

∴ BD = 2 × BO

= 2 × 2 = 4 cm

S26. Ans.(a)

Sol.

According to question

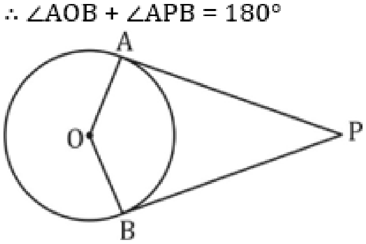
Given: PAOB is quadrilateral

∵ ∠AOB : ∠APB

5x : 1x

Note: In Quadrilateral sum of opposite angle is 180°

∴ ∠AOB + ∠APB = 180°



Then 5x + x = 180°

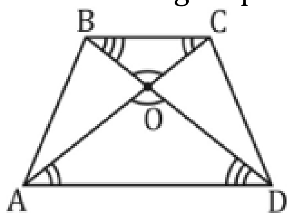
6x = 180°

x = 30°

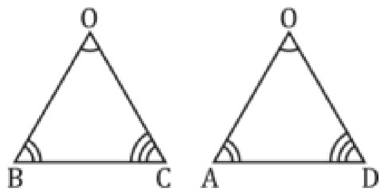
∴ ∠APB = 30°

S27. Ans.(d)

Sol. According to question,



$\Delta AOD \sim \Delta COB$



$\therefore \frac{OB}{OD} = \frac{OC}{OA}$

$\Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$

$\Rightarrow 9x - 57 = x^2 - 8x + 15$

$\Rightarrow x^2 - 17x + 72 = 0$

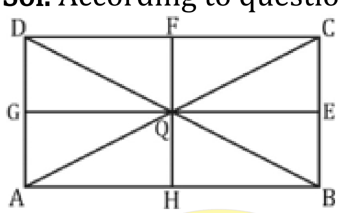
$\Rightarrow x(x-8) - 9(x-8) = 0$

$\Rightarrow (x-8)(x-9) = 0$

$\Rightarrow x = 8$ or 9

S28. Ans.(a)

Sol. According to question,



Given:

QA = 3 cm

QB = 4 cm

QC = 5 cm

QD = ?

As we know that

$\Rightarrow QD^2 + QB^2 = QA^2 + QC^2$

$\Rightarrow QD^2 + (4)^2 = (3)^2 + (5)^2$

$\Rightarrow QD^2 + 16 = 9 + 25$

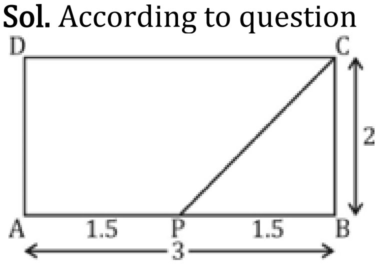
$\Rightarrow QD^2 = 34 - 16$

$\Rightarrow QD^2 = 18$

$QD = \sqrt{18}$, $QD = 3\sqrt{2}$

S29. Ans.(d)

Sol. According to question



$\Rightarrow$ In ΔCBP
$\Rightarrow CP^2 = BP^2 + BC^2$
$\Rightarrow CP^2 = (1.5)^2 + (2)^2$
$\Rightarrow CP^2 = 2.25 + 4$
$\Rightarrow CP^2 = 6.25$
$\Rightarrow CP = \sqrt{6.25}$
$\Rightarrow CP = 2.5$

$\therefore \sin \angle CPB = \frac{BC}{CP}$

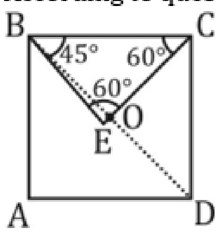
$\sin \angle CPB = \frac{2}{2.5}$

$\sin \angle CPB = \frac{4}{5}$

S30. Ans.(b)

Sol.

According to question



ABCD is a square and BCE is an equilateral triangle
$\therefore \angle CEB = 60°$
If BD is a diagonal
$\therefore \angle CBD = 45°$
Then In ΔBOC
$\angle CBO + \angle BOC + \angle BCE = 180°$
$\angle BOC = 180° - 60° - 45°$
$\angle BOC = 75°$