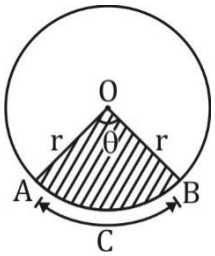
Circle Notes for SSC CGL and CHSL

➢ **Circle :→**

Sector :-



$\Rightarrow \theta = \frac{l \rightarrow \text{length of arc AB}}{r \rightarrow \text{radius}}$

↓

always in Radian

$I^C = \frac{180°}{\pi}$

$\pi^C = 180°$

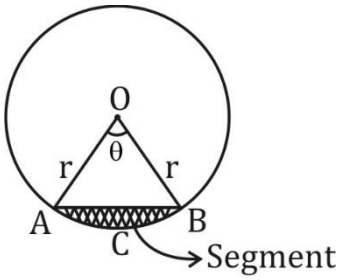
$I° = \frac{\pi^C}{180}$

$\Rightarrow$ Length of arc $= 2\pi r\frac{\theta}{360°}$

$\Rightarrow$ Area of sector OAB $= \pi r^2 \frac{\theta}{360°}$

$\Rightarrow$ Perimeter of sector $= \pi r\frac{\theta}{180°} + 2r$

Segment :-



→ Area of segment = area of sector OACB – area of ΔOAB

$= \pi r^2 \frac{\theta}{360°} - \frac{1}{2}r^2 \sin\theta$

→ Perimeter = length of arc ACB + Chord length AB

$= (2\pi r)\frac{\theta}{360°} + 2r\sin\left(\frac{\theta}{2}\right)$

Q1. Find the area of a segment of a circle with a central angle of 120 degrees and a radius of 8 cm.

Sol. Area of segment = $\pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$

$= \pi(8)^2 \frac{120^\circ}{360^\circ} - \frac{1}{2}(8)^2 120^\circ$

$= 83.047$

Q2. Find the area of a sector with an arc length of 30 cm and a radius of 10 cm.

Sol. Length of arc $= 2\pi r \frac{\theta}{360^\circ} = 30$

$\pi r \frac{\theta}{360^\circ} = 15$

Area of sector OAB $= \pi r^2 \frac{\theta}{360^\circ} = (\pi r \frac{\theta}{360^\circ})\ r = 15 \times 10 = 150$ cm

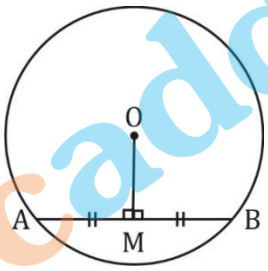
Q3. In a circle of radius 21 cm and arc subtends an angle of 72 at centre. The length of arc is?

Sol. Length of arc $= 2\pi r \frac{\theta}{360^\circ}$

$= 2\ \pi\ \times 21\ \times \frac{72^\circ}{360^\circ} = 26.4$ cm

<u>Important Properties Of Circle : -</u>

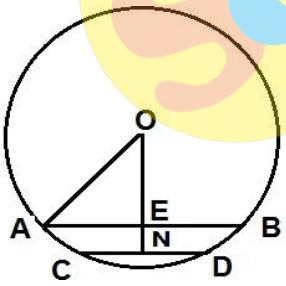
- Perpendicular from the centre of a circle to a chord bisects the chord.



AM = MB

Q1. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the center of the circle. The distance between them is 1 cm. Find the length of the radius?

Sol.



Let ON = x , AO = r

In triangle AOE

$r^2 = 16 + (x-1)^2$

In triangle OCN

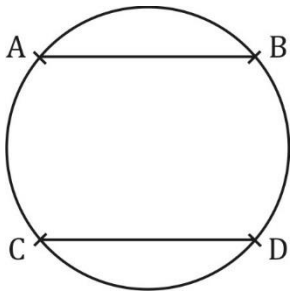
$r^2 = 9 + x^2$

$16 + (x-1)^2 = 9 + x^2$

$x = 4$

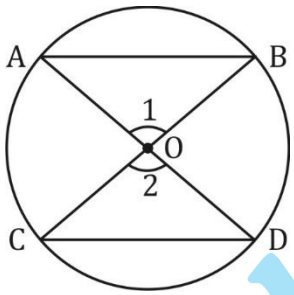
$r^2 = 9 + 16$, $r = 5$ cm

- Chords corresponding to equal arcs are equal.



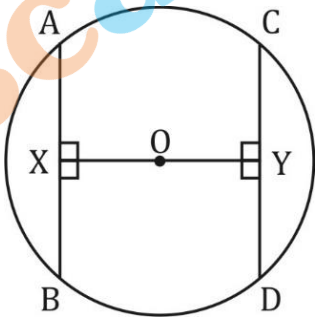
If $\overset{\frown}{AB} = \overset{\frown}{CD}$, then chord , AB = CD

- Equal Chords of Circle Subtends equal angles at the centre.



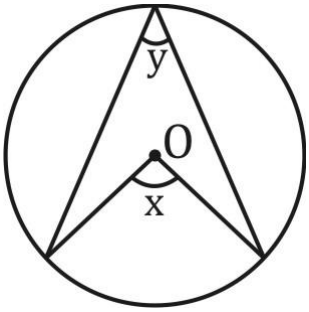
If AB = CD
then ∠1 = ∠2

- Equal chords of a circle are equidistance from the centre.



If AB = CD, Then OX = OY

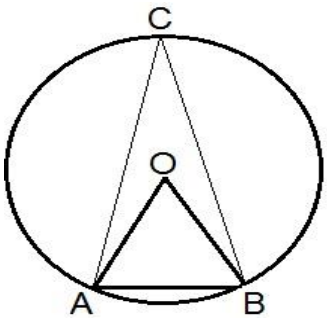
- The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



x = 2y

Q1. The length of chord of a circle is equal to the radius of the circle .The angle which this chord subtends in the major segment of the circle is equal to?

Sol.



OA = OB = r

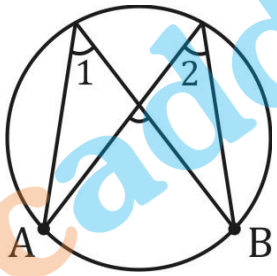
AB is equal to radius

Therefore triangle OAB is an equilateral triangle

Angle OAB = 60°

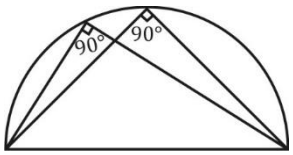
Angle ACB, angle which chord subtends at major angle = $\frac{60^\circ}{2}$ = 30°

- Angle in same segment of a circle are equal.



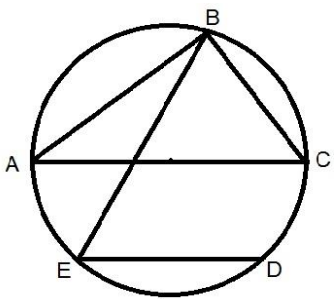
∠1 = ∠2

- Angle in a semicircle is always a right angle.



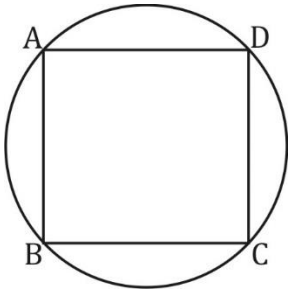
Q1. AC is the diameter of a circumcircle of triangle ABC. Chord ED is parallel to the diameter AC. If Angle CBE = 50°, then the measure of angle DEC is?

Sol.



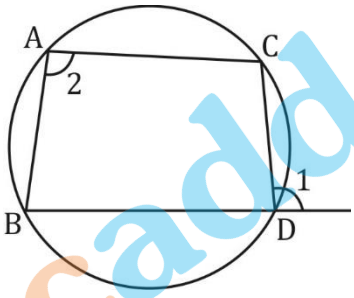
Angle CBE = 50°
Angle ABC = 90° (Angle in a semicircle is always a right angle)
Angle ABE = 90° - 50° = 40°
Angle ABE = Angle ACE = 40°
Angle ACE = Angle CED = 40° (Alternate Angles)

- If, ABCD is a cyclic quadrilateral



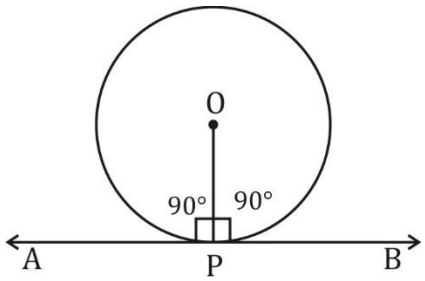
∠A + ∠C = 180°
∠B + ∠D = 180°

- ABCD is a cyclic quadrilateral



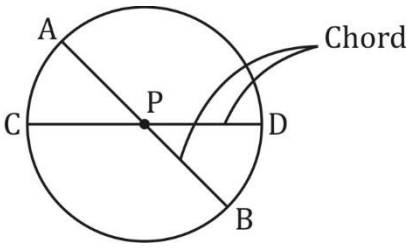
∠1 = ∠2

- A tangent at any point of circle is Perpendicular to the radius through the point of contact



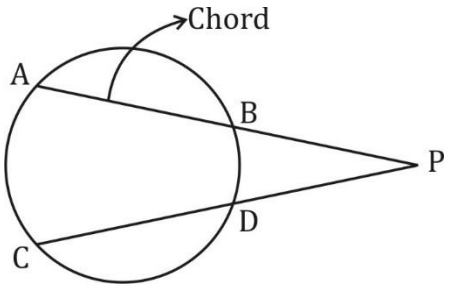
OP⊥AB

-



PA × PB = PC × PD

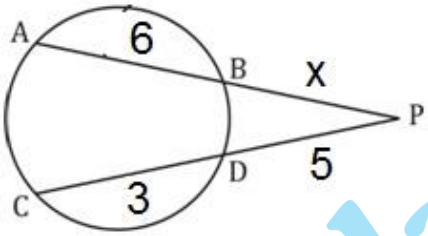
➢



PA × PB = PC × PD

Q1. Chords AB and CD of a circle intersects externally at P. If AB = 6 cm, CD = 3 cm and PD = 5 cm, then the length of PB is?

Sol.



PA × PB = PC × PD

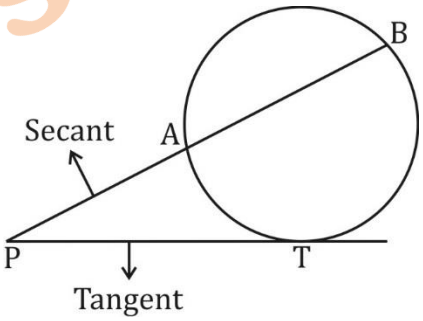
x(6+x)= 5 × 8

$x^2 + 6x - 40 = 0$

x = 4 , -10

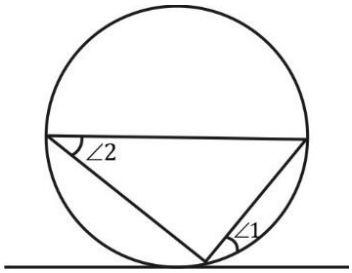
PB = 4

➢



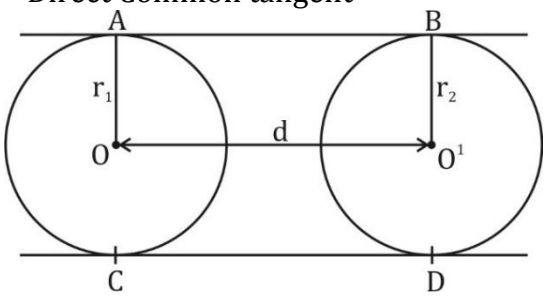
$PT^2 = PA \times PB$

➢



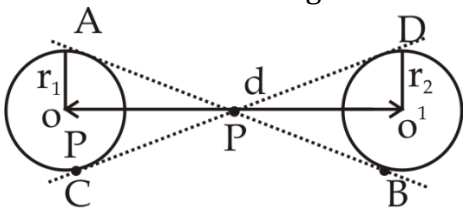
∠1 = ∠2

- AB = CD = Direct Common tangent



$AB = CD = \sqrt{d^2 - (r_1 - r_2)^2}$

- AB = CD Transverse Common Tangents



$AB = CD = \sqrt{d^2 - (r_1 + r_2)^2}$

Q1. If the radii of two circles be 6 cm and 3 cm and the length of transverse common tangent be 8 cm, then the distance between the two centers is?

Sol. Length of transverse Common Tangent $= \sqrt{d^2 - (r_1 + r_2)^2}$

$8 = \sqrt{d^2 - (6+3)^2}$

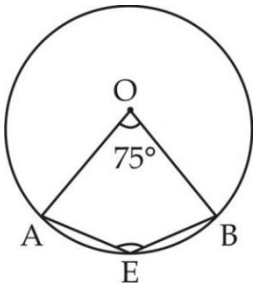
$64 = d^2 - 81$

$d^2 = 145$

$d = \sqrt{145}$ cm

Questions

Q1. In the given figure, O is the centre of the circle and ∠AOB = 75°, then ∠AEB will be?



(a) 142.5
(b) 162.5
(c) 132.5
(d) 122.5

Q2. In a circle, center angle is 120°. Find the ratio of a major angle and minor angle?

(a) 2:7
(b) 2:1
(c) 2:9
(d) 2:3

Q3. A, B & C are three points on a circle such that a tangent touches the circle at A and intersects the extended part of chord BC at D. Find the central angle made by chord BC, if angle CAD = 39°, angle CDA = 41°?

(a) 122
(b) 123
(c) 132
(d) 142

Q4. Find the length of the common tangent of two externally touch circle with radius 16 cm and & 9 cm respectively?

(a) 12 cm
(b) 24 cm
(c) 13cm
(d) 28 cm

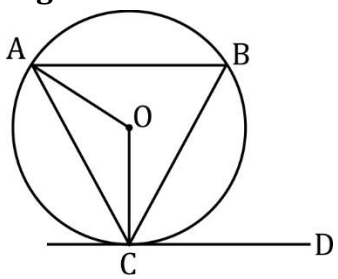
Q5. ABC is an isosceles triangle a circle is such that it passes through vertex C and AB acts as a tangent at D for the same circle. AC and BC intersects the circle at E and F respectively AC = BC = 4 cm and AB = 6 cm. Also, D is the mid-point of AB. What is the ratio of EC : (AE + AD)?

(a) 9:7
(b) 3:4
(c) 4:3
(d) 1:3

Q6. The radius of two concentric circles are 17 cm and 10 cm. A straight line ABCD intersects the larger circle at the point A and D and intersects the smaller circle at the points B and C. If BC = 12 cm, then the length of AD (in cm) is?

(a) 20
(b) 24
(c) 30
(d) 34

Q7. In the given diagram O is the centre of the circle and CD is a tangent. Angle CAB and angle ACD are supplementary to each other and angle OAC = 30°. Find the value of angle OCB?



(a) 30°
(b) 20°
(c) 60°
(d) None of the above

Q8. Two circle of equal radius of 'r' passes through centre of each other. Find the length of common tangent.

(a) 3r

(b) $\sqrt{5}$ r

(c) $\sqrt{3}$ r

(d) 2r

Q9. Two circle of radius 9 cm and 4 cm and distance between their centre is 13 cm. Find the length of direct common tangent of the circles.

(a) 11 cm

(b) 12 cm

(c) 10 cm

(d) 8 cm

Q10. PT is a tangent of circle. AB is a chord of circle if AB = 18 cm and PT = 2 AP. Find length of PT.

(a) 6 cm

(b) 9 cm

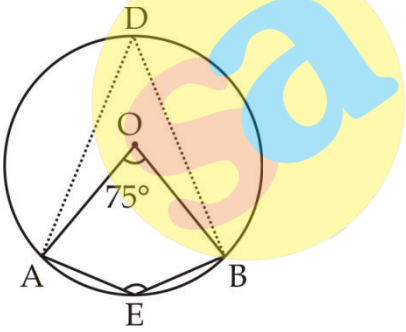
(c) 8 cm

(d) 12 cm

Solutions

S1. Ans.(a)

Sol.



$\angle AOB = 75°$

$\angle ADB = \frac{\angle AOB}{2}$ [Center angle of a circle is twice the angle of the major arc]

$= \frac{75°}{2} = 37.5°$

AEBD is a cyclic quadrilateral then,

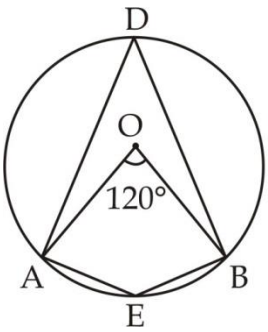
$\angle E + \angle D = 180°$

$\angle E + 37.5° = 180°$

$\angle E = 142.5°$

S2. Ans.(b)

Sol.



$\angle ADB = \frac{\angle O}{2} = \frac{120°}{2} = 60°$

[Center angle of a circle is twice the angle of the major arc]

AEBD is a cyclic quadrilateral then,

$\angle AEB + \angle ADB = 180°$

$\angle AEB + 180° - 60°$

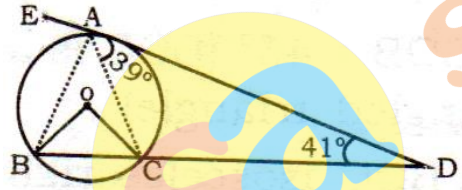
$\angle AEB = 120°$

Required ratio = major angle : minor angle

$= 120° : 60° = 2 : 1$

S3. Ans.(a)

Sol.



$\angle ACB = \angle CAD + \angle CDA$ [Sum of two interior angle is equal to opposite of exterior angle]

$\angle ACB = 39° + 41° = 80°$

$\angle BAE = \angle BCA = 80°$

[Alternate segment]

$\angle EAB + \angle BAC + \angle CAD = 180°$

[Linear angle]

$80° + \angle BAC + \angle CAD = 180°$

$\angle BAC = 61°$

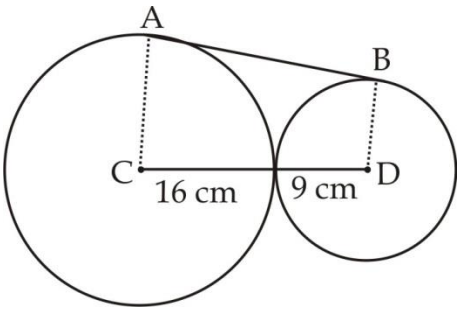
$\therefore \angle BOC = 2 \times \angle BAC$

[Center angle is twice the angle subtended by the major arc]

$= 2 \times 61° = 122°$

S4. Ans.(b)

Sol.



Length of common tangent = (Distance between two circle)^2–(Radius$_1$ – Radius$_2$)^2

$AB^2 = CD^2 - (16 - 9)^2$

$AB^2 = (16+9)^2 - (7)^2$

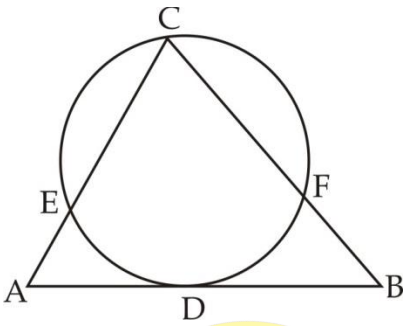
$AB^2 = 625 - 49$

$AB^2 = 576$

$AB = 24$ cm

S5. Ans.(d)

Sol.



Here, AC and BC are the secants of the circle and AB is tangent at D

$AD = DB = \frac{6}{2}$

$\therefore AE \times AC = AD^2$

$AE \times 4 = (3)^2 \Rightarrow AE = \frac{9}{4}$

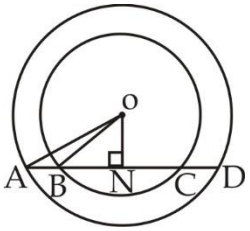
$\therefore CE = 4 - \frac{9}{4} = \frac{7}{4}$

$\therefore CE : (AE + AD)$

$= \frac{7}{4} : \left(\frac{9}{4} + 3\right) = \frac{7}{4} : \frac{21}{4} = 1 : 3$

S6. Ans.(c)

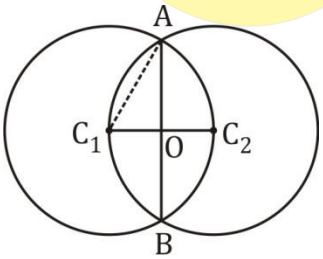
Sol. According to question



Given :
BC = 12cm, OA = 17 cm
OB = 10 cm
∴ BN = NC = 6 cm
∴ In right angle ΔONB
$OB^2 = ON^2 + BN^2$
$(10)^2 = ON^2 + (6)^2$
$ON^2 = 100 - 36$
$ON^2 = 64$
ON = 8 cm
In right angle ΔONA
$OA^2 = ON^2 + AN^2$
$(17)^2 = (8)^2 + AN^2$
$AN^2 = 289 - 64$
$AN^2 = 225$
AN = 15 cm
AD = 2 × AN
∴ AD = 15 × 2 = 30 cm

S7. Ans.(a)
Sol. ∠CAB and ∠ACD are supplementary
∴∠CAB + ∠ACD = 180°
And AB || CD
∴∠DCB = ∠ABC
∠OAC = ∠OCA = 30°
∴∠AOC = 120°⇒∠ABC = 60°
⇒∠DCB = ∠ABC = 60°
∠OCD = 90°
∠OCB = ∠OCD - ∠DCB
∠OCB = 90°– 60°
∠OCB = 30°

S8. Ans.(c)
Sol.



AB = common tangent
C_1andC_2 are centres of circle with radius 'r'.
$C_1A = r$
$C_1C_2 = r$
$C_1O = \frac{r}{2}$

$AO = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}r}{2}$

AB = 2AO

AB = $\sqrt{3}r$

S9. Ans.(b)

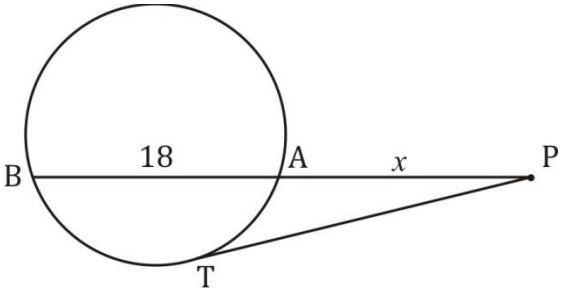
Sol. Direct common tangent = $\sqrt{d^2 - (R-r)^2}$

$= \sqrt{(13)^2 - (9-4)^2}$

$= 12$ cm

S10. Ans.(d)

Sol.



Let AP = x

PT = $2x$

$PT^2 = PA \times PB$

$(2x)^2 = x(18 + x)$

$4x^2 = 18x + x^2$

$3x^2 - 18x = 0$

$3x(x - 6)$

$x = 6$

∴ PT = 6 × 2 = 12 cm