Quant Mega Quiz for SSC CGL – Advance Level (Solutions)

S1. Ans.(b)

Sol.

$x^{y+z} = 1, \quad y^{x+z} = 1024 \quad z^{x+y} = 729$

$\downarrow \quad \downarrow$

$y^{x+z} = 2^{10} \quad z^{x+y} = 9^3$

$\downarrow \quad \downarrow$

from here $\quad z = 9, x = 1, y = 2$

$y = 2, x + z = 10$

So, value of $(z+1)^{y+x+1}$

$\Rightarrow (9+1)^{2+1+1}$

$\Rightarrow 10000$

S2. Ans.(a)

Sol. N = $(12345)^2$ + 12345 + 12346

Let x = 12345

N = $(x)^2 + x + (x + 1)$

N = $x^2 + 2x + 1$

N = $(x + 1)^2$

$\sqrt{N} = x + 1 \Rightarrow 12345 + 1 \Rightarrow \boxed{12346}$

S3. Ans.(b)

Sol.

METHOD 1

$\sqrt{(1-p^2)(1-q^2)} = \frac{\sqrt{3}}{2}$

Let p = 0, $\quad q = \frac{1}{2}$

Satisfying $\left(\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}\right)$

$= \left(\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\right)$

Then find $\sqrt{2p^2 + 2q^2 + 2qp} + \sqrt{2p^2 + 2q^2 - 2pq}$

$\sqrt{2 \times \frac{1}{4} + 0} + \sqrt{2 \times \frac{1}{4} - 0}$

$2\sqrt{\frac{1}{2}} = \sqrt{2}$

METHOD – 2

$\sqrt{(1-P^2)(1-q^2)} = \frac{\sqrt{3}}{2}$

Squaring both sides

$1 - p^2 - q^2 + p^2q^2 = \frac{3}{4}$

$\Rightarrow p^2 + q^2 = \frac{1}{4} + p^2q^2$

$\therefore p^2 + q^2 + pq = \frac{1}{4} + p^2 q^2 + pq = (\frac{1}{2} + pq)^2$

& $p^2 + q^2 - pq = p^2q^2 + \frac{1}{4} - pq = (\frac{1}{2} - pq)^2$

$\therefore \sqrt{2p^2 + 2q^2 + 2pq} + \sqrt{2p^2 + 2q^2 - 2pq}$

$\Rightarrow \sqrt{2}\left[\left(\frac{1}{2} + pq\right) + \left(\frac{1}{2} - pq\right)\right] = \sqrt{2}$

S4. Ans.(c)

Sol.

$A = 1 + 2^P, B = 1 + 2^{-P}$

$B = 1 + \frac{1}{2^P}$

$B = \frac{2^P + 1}{2^p}$

$B = \frac{A}{A - 1}$

<u>Alternate method:</u>

Put P = 1

A = 3, B = $\frac{3}{2}$

Now check option (c)

$\Rightarrow \frac{A}{A-1} \Rightarrow \boxed{\frac{3}{2} = B}$ Satisfying.

S5. Ans.(c)

Sol. Given, a cos θ – b sin θ = c

On squaring both sides, we get

$a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \cos \theta \sin \theta = c^2$

$\Rightarrow a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \sin \theta \cos \theta = c^2$

$\Rightarrow a^2 + b^2 - c^2 = a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta$

$\Rightarrow (a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a \sin \theta + b \cos \theta = \pm\sqrt{a^2 + b^2 - c^2}$

S6. Ans.(b)

Sol.

$\frac{\sin^6 \theta - \cos^6 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{(\sin^2 \theta)^3 - (\cos^2 \theta)^3}{\sin^2 \theta - \cos^2 \theta}$

$= \frac{(\sin^2 \theta - \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta + \sin^2 \theta \cos^2 \theta)}{\sin^2 \theta - \cos^2 \theta}$

$= \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta$

$= (\sin^2 \theta + \cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta = 1 - \sin^2 \theta \cos^2 \theta$

S7. Ans.(c)

Sol. Here, p = a sin x + b cos x and q = a cos x – b sin x

On squaring both sides,

$\Rightarrow p^2 = a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x$ ...(i)

q = a cos x – b sin x

On squaring both sides,

And $q^2 = a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x$ ...(ii)

Now, add equation (i) and equation (ii), we get

$\therefore p^2 + q^2 = a^2 (\sin^2 x + \cos^2 x) + b^2 (\cos^2 x + \sin^2 x$

$= a^2 + b^2$

S8. Ans.(d)

Sol. (sin x. cos y + cos x. sin y) (sin x. cos y – cos x. sin y)

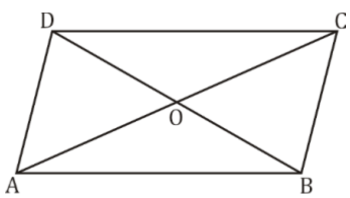
= sin (x + y). sin (x – y)

$= \sin^2 x - \sin^2 y$

$[\because \sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)]$

S9. Ans.(b)

Sol.



I. ABCD is a parallelogram, then

$AC^2 + BD^2 = 2(AB^2 + BC^2)$

So it is not true.

II. ABCD is a rhombus and diagonals AC and BD bisect each other.

∴ AO = OC

and OB = OD

In ΔAOB,

$(4)^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

$\therefore AC^2 + BD^2 = 64$

$= (4)^3$ i.e., n^3

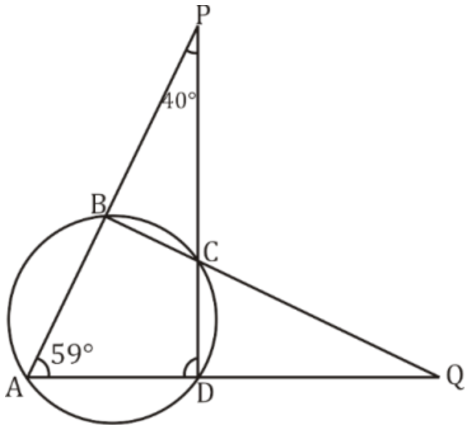
So only II is true.

S10. Ans.(c)

Sol. Given ∠PAQ = 59°

and ∠APD = 40°

In ΔADP



∠ADP = 180° – 59° – 40° = 81°

∠ADC + ∠ABC = 180° (cyclic quadrilateral)

∠ABC = 180° – 81° = 99°

Now in ΔABQ

$\angle ABQ + \angle BAQ + \angle AQB = 180°$

$\therefore \angle AQB = 180°$ – (99° + 59°)

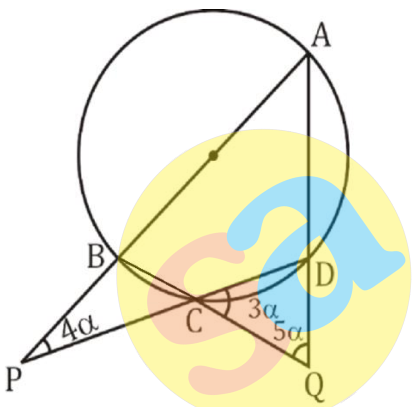
= 180° – 158° = 22

S11. Ans.(b)

Sol.

Given $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = \alpha$ (say)

$\therefore x = 3\alpha, y = 4\alpha \ and \ z = 5\alpha$



Since,

$\angle DCQ = \angle BCP = 3\alpha$

(Vertically opposite angle)

In ΔDCQ, $\angle CDQ = 180°$ – $(3\alpha + 5\alpha) = 180°$– 8α

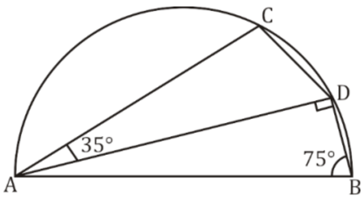
$\angle QDC = \angle CBA = 180°$ – $8\alpha \Rightarrow \angle PBC = 8\alpha$, By the proportion of cyclic quadrilateral,

In ΔPBC,

$\angle P + \angle B + \angle C = 180°$

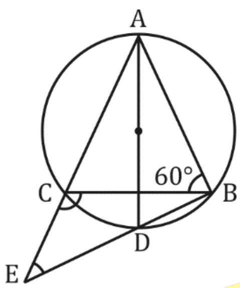
$$\therefore 4\alpha + 8\alpha + 3\alpha = 180° \Rightarrow \alpha = \frac{180°}{15} \Rightarrow \alpha = 12°$$

$x = 36°, y = 48°, z = 60°$

S12. Ans.(a)
Sol.


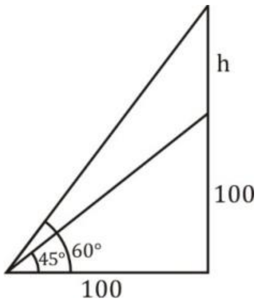
Since, ΔADB is a right angled triangle at D.
$\therefore \angle DAB = 180° - (90° + 75°)$
$\Rightarrow \angle DAB = 15°$
Also, ABDC is cyclic quadrilateral.
$\therefore \angle CAB + \angle BDC = 180°$
$\Rightarrow \angle BDC = 180° - (35° + 15°) = 130°$

S13. Ans.(b)
Sol. $\angle AQP = \frac{1}{2} \times \angle AOP = \frac{75°}{2} = 37.5°$

S14. Ans.(a)
Sol.


Here, AD is the angle bisector of ∠A of equilateral ΔABC
∴ It passes through centre of circle O and AD is the diameter
∴∠ABD = 90° (angle in a semi-circle) and ∠CBE = 90° – 60° = 30°
Now,
∠ECB = 180° – 60° = 120°
(∴ ACB = 60°, angle of equilateral triangle)
Then, ∠CEB = 180° – 120° – 30°
= 30°

S15. Ans.(d)
Sol. If the angles made by the all three points are same then the tower must be lying at a point which has same distance from all the points ie; Tower must be at the Circum centre
Circum radius of the triangle = $\frac{a}{\sqrt{3}}$
$\tan\alpha = \frac{x}{\frac{a}{\sqrt{3}}}$
$x = \frac{a}{\sqrt{3}} \tan\alpha$

S16. Ans.(c)
Sol.


$\tan 60° = \frac{h + 100}{100}$
$100\sqrt{3} = h + 100$
$h = 100(\sqrt{3} - 1)m$

S17. Ans.(d)
Sol.
$a = \sqrt{7 + 2 \times \sqrt{4} \times \sqrt{3}}$
$= \sqrt{(2 + \sqrt{3})^2}$
$= (2 + \sqrt{3})$
Similarly, $b = (2 - \sqrt{3})$
ab = 1
then $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
$= (4)(7 + 2\sqrt{12} + 7 - 2\sqrt{12} - 1)$
$= 4 \times (13)$
$= 52$

S18. Ans.(c)
Sol. Given,
For $(a + 2)^2 = a^2 + 4a + 4$
$(a^2 + 4a + 4) - (a + 1) = 0$
$(a + 2)^2 - (a + 2) = -1$
On dividing by (a + 2) from both sides,
$(a + 2) - 1 = -\frac{1}{(a + 2)}$
$(a + 2) + \frac{1}{(a + 2)} = 1$
(If $x + \frac{1}{x} = 1$; $x^3 = -1$)
Hence $(a + 2)^3 = -1$
$a^3 + 12a + 6a^2 + 8 = -1$
$a^3 + 12a + 6a^2 + 10 = 1$

S19. Ans.(a)

Sol. $\frac{1}{2}AB \times BE = 7$

$AB^2 = 7 \times 2 = 14$

$AB = \sqrt{14}$

$\because AB = BE$

$EC = 3\sqrt{14}$

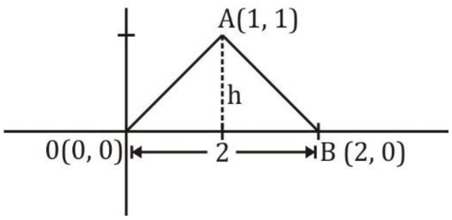
$BC = BE + EC = 4\sqrt{14}$ cm

Area of rectangle FECD = $3\sqrt{14} \times \sqrt{14} = 42\ cm^2$

Area of shaded region = $\frac{\text{Area of rectangle FECD}}{2} = 21\ cm^2$

S20. Ans.(b)

Sol. Let a = 0 Now find the area of triangle with vertices (0, 0), (1, 1) & (2, 0)



Area of triangle = $\frac{1}{2}$ Base × height

$= \frac{1}{2} \times 2 \times 1 = 1\ cm^2$

S21. Ans.(c)

Sol. $\angle CAD + \angle ACD = \angle CDB$

$2x = y$

$x + 180 - 2y + 96 = 180$

$2y - x = 96$

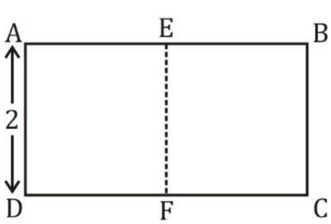
$4x - x = 96$

$x = 32°$

$\angle DBC = y = 64°$

S22. Ans.(b)

Sol.



Let $\frac{AB}{AD} = \frac{x}{y}$

Also, $\frac{AD}{AE} = \frac{x}{y}$

$$\frac{2AD}{AB} = \frac{x}{y}$$

$$2\frac{y}{x} = \frac{x}{y}$$

$2y^2 = x^2$

$x = \sqrt{2}\,y$

given, y = 2 cm

$x = 2\sqrt{2}$ cm

Area of rectangle ABCD = $4\sqrt{2}$ cm^2

Area of smaller rectangle = $2\sqrt{2}$ cm^2

S23. Ans.(a)

Sol.

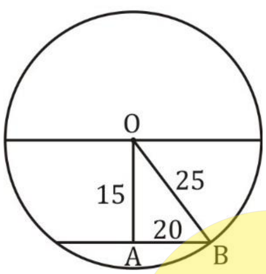
Area of triangle = $\frac{\sqrt{3}}{4} \times (2r)^2 = \sqrt{3}\, r^2$

Area of circular sectors = $\frac{\pi}{2} \times r^2$

Area of shaded region = $r^2\,(2\sqrt{3} - \pi)/2$

S24. Ans.(d)

Sol.



Radii of the circular section = $\sqrt{25^2 - 15^2}$ = 20 cm

Circumference of the plane circular section ⇒ 2πr ⇒ 40π

S25. Ans.(c)

Sol. Volume of sphere = volume of cone

$$\frac{4}{3}\pi \times 14 \times 14 \times 14 = \frac{1}{3}\pi \times 21 \times 21 \times h$$

16 × 14 = 9 h

$\frac{224}{9}$ = h