SSC CGL Mains Mega Quiz (Quant)_Solution

S1. Ans.(c)
Sol. We can not determine the maximum value of given function.

S2. Ans.(b)
Sol.

$$\frac{\sin(y-z)+\sin(y+z)+2\sin y}{\sin(x-z)+\sin(x+z)+2\sin x}$$

$$\Rightarrow \frac{\sin y\cos z-\cos y\sin z+\sin y\cos z+\cos y\sin z+2\sin y}{\sin x\cos z-\cos x\sin z+\sin x\cos z+\cos x\sin z+2\sin x}$$

$$\Rightarrow \frac{2\sin y\cos z+2\sin y}{2\sin x\cos z+2\sin x} \Rightarrow \boxed{\frac{\sin y}{\sin x}}$$

Alternate Method

options are independent of z

so put z = 0

hence equation becomes $= \boxed{\frac{\sin y}{\sin x}}$

S3. Ans.(b)
Sol.

$$\frac{\sin(x+y)-2\sin x+\sin(x-y)}{[\cos(x-y)+\cos(x+y)-2\cos x]} \times \frac{\sin 10x-\sin 8x}{\cos 10x+\cos 8x}$$

$$\Rightarrow \frac{2\sin x\cos y-2\sin x}{2\cos x\cos y-2\cos x} \times \frac{\sin 10x-\sin 8x}{\cos 10x+\cos 8x}$$

$(\sin A-\sin B=\cos(A+B)/2 .\sin(A-B)/2 \ \&\cos A+\cos B=\cos(A+B)/2.\cos(A-B)/2)$

$$\Rightarrow \frac{2\sin x\cos y-2\sin x}{2\cos x\cos y-2\cos x} \times \frac{2\cos 9x\times\sin x}{2\cos 9x\times\cos x}$$

$$\Rightarrow \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \boxed{\tan^2 x}$$

S4. Ans.(b)
Sol.

By Hit and Trial

$b^2x^2 - a^2y^2 = a^2b^2$ ...(i)

Now, putting the value of x and y in Eq. (i), we get

$\Rightarrow b^2(a\sec\theta)^2 - a^2(b\tan\theta)^2 = a^2b^2$

$\Rightarrow b^2a^2\sec^2\theta - a^2b^2\tan^2\theta = a^2b^2$

$\Rightarrow a^2b^2(\sec^2\theta - \tan^2\theta) = a^2b^2$

$\therefore a^2b^2 = a^2b^2$

$[\because \sec^2\theta - \tan^2\theta = 1]$

Hence, option (b) is the right answer.

S5. Ans.(b)
Sol.

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\times 2\sin\frac{(x-y)}{2}\times\cos\frac{(x+y)}{2}}{2\cos\frac{(x+y)}{2}.\cos\left(\frac{(x-y)}{2}\right).2\sin\frac{(x+y)}{2}\sin\frac{(x-y)}{2}}$$

$\Rightarrow 1$

Method-2

$$\frac{[(\sin x+\sin y)(\sin x-\sin y)]}{[(\cos x+\cos y)(\cos y-\cos x)]}$$

$$\Rightarrow \frac{\sin^2 x-\sin^2 y}{\cos^2 y-\cos^2 x}$$

$$\Rightarrow \frac{(\sin^2 x-\sin^2 y)}{[-(1-\sin^2 x)+(1-\sin^2 y)]}$$

$$\Rightarrow \frac{\sin^2 x-\sin^2 y}{(\sin^2 x-\sin^2 y)}$$

$= 1$

S6. Ans.(b)
Sol.

$3\sin^2\phi + 4\cos^2\phi$

or $3\sin^2\phi + 3\cos^2\phi + \cos^2\phi$

$= 3 + \cos^2\phi$ ($\because$ maximum value of $\cos^2\phi = 1$)

$= 3 + 1 = 4$

S7. Ans.(b)
Sol.

$$\Rightarrow \frac{\frac{\sin 5\theta}{\cos 5\theta}+\frac{\sin 3\theta}{\cos 3\theta}}{4\cos 4\theta\left(\frac{\sin 5\theta}{\cos 5\theta}-\frac{\sin 3\theta}{\cos 3\theta}\right)}$$

$$\Rightarrow \frac{\sin 5\theta\cos 3\theta+\sin 3\theta\cos 5\theta}{4\cos 4\theta(\cos 3\theta\sin 5\theta-\sin 3\theta\cos 5\theta)}$$

$$\Rightarrow \frac{\sin 2\times 4\theta}{4\cos 4\theta.\cos 2\theta}$$

$$\Rightarrow \frac{2\sin 4\theta\cos 4\theta}{4\cos 4\theta.\sin 2\theta} \Rightarrow \frac{2\times 2\sin 2\theta.\cos 2\theta}{4\sin 2\theta}$$

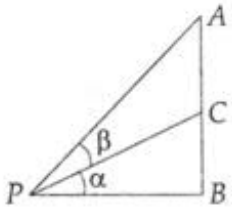
$\Rightarrow \cos 2\theta$

S8. Ans.(c)
Sol.

Given: Vertical pole = AB; Middle point of AB = C or AC = $\frac{AB}{2}$; Angle $\angle APC = \beta$ and BP = n AB.

Let $\angle CPB = \alpha$.

Therefore $\angle APB = \alpha + \beta$.



We know that in ΔCPB, $\tan\alpha = \frac{CB}{BP} = \frac{CB}{nAB} = \frac{AB}{2nAB} = \frac{1}{2n}$.

Similarly, in ΔAPB, $\tan(\alpha + B) = \frac{AB}{BP} = \frac{AB}{nAB} = \frac{1}{n}$.

We also know that $\beta = \alpha + \beta - \alpha$ or $\tan\beta = \tan\{(\alpha + \beta) - \alpha\}$

$$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\tan\alpha} = \frac{\frac{1}{n} - \frac{1}{2n}}{1 + \frac{1}{n} \times \frac{1}{2n}} = \frac{n}{2n^2 + 1}$$

$$\because \left\{\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right\}$$

S9. Ans.(d)

Sol.

$$\text{Given, } \tan\theta = \frac{3}{4} = \frac{p}{b}$$

Then,

$$h = \sqrt{p^2 + b^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\therefore \sin\theta = \frac{p}{h} = \frac{3}{5}$$

$$\cos\theta = \frac{b}{h} = \frac{4}{5}$$

Now,

$$25x \sin^2\theta \cos\theta = \tan^2\theta$$

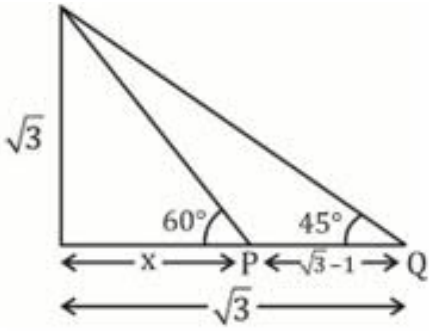
$$\Rightarrow 25.x.\left(\frac{3}{5}\right)^2.\frac{4}{5} = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow 25.x.\frac{9}{25}.\frac{4}{5} = \frac{9}{16}$$

$$\therefore x = \frac{5}{64}$$

S10. Ans.(a)

Sol.



Let height of light house=$\sqrt{3}$

Tan45= height/base

Base=$\sqrt{3}$

Tan 60=$\frac{\sqrt{3}}{x}$

x=1 ,therefore,

$(\sqrt{3}-1)$distance $\rightarrow 60\times 4\,(\sqrt{3}-1)$

$1 \rightarrow 240$

HENCE,(height) $\sqrt{3} \rightarrow 240\sqrt{3}$

S11. Ans.(a)

Sol.

$$\frac{1}{sin^4(90-\theta)}+\frac{1}{[\cos^2(90-\theta)]-1}$$

$$=\frac{1}{\cos^4\theta}+\frac{1}{\sin^2\theta-1}$$

$$=\frac{1}{\cos^4\theta}-\frac{cos^2\theta}{\cos^4\theta}$$

$$=\frac{sin^2\theta}{\cos^4\theta}=\tan^2\theta sec^2\theta$$

Alternate (putting method)

Put $\theta = 45$

$$=\frac{1}{sin^4 45}+\frac{1}{\cos^2 45-1}$$

$$=2\times 2-\frac{1}{\frac{1}{2}-1}$$

$$=4-2=(2)$$

Option (1) Satisfying = $1\times\left(\sqrt{2}\right)^2=2$

S12. Ans.(c)

Sol.

$$\sin\theta+\sin^2\theta+\sin^3\theta=1$$

$$\sin\theta+\sin^3\theta=\cos^2\theta$$

$$\sin\theta\,(1+\sin^2\theta)=\cos^2\theta$$

$$\sin^2\theta\,(1+\sin^2\theta)^2=\cos^4\theta$$

$$(1-\cos^2\theta)\left(1+(1-\cos^2\theta)\right)^2=\cos^4\theta$$

$$(1-\cos^2\theta)(2-\cos^2\theta)^2=\cos^4\theta$$

$$(1-\cos^2\theta)(4+\cos^4\theta-4\cos^2\theta)=\cos^4\theta$$

$$4+\cos^4\theta-4\cos^2\theta-4\cos^2\theta-\cos^6\theta+4\cos^4\theta=\cos^4\theta$$

$$\cos^6\theta-4\cos^4\theta+8\cos^2\theta=4$$

S13. Ans.(c)
Sol.

$$\frac{[tan\ (90-A) + cot(90-A)]^2}{2\ Sec^2\ (90-2A)}$$

Put A = 30 or 45^0

$= \frac{[1+1]^2}{2} = 2$

S14. Ans.(a)
Sol.

$Sin\ (90 - x)cos[\pi - (x - y)] + cos(90 - x)\ \sin[\pi - (y - x)]$

Put $x = y = 45$

Sin 45 × -cos O + cos45 × O

$= \frac{1}{\sqrt{2}}$ = Option (1)

= - Cos y

Alternate method

−cos x cos (x-y) -sin x(sin (x-y)] as(cos a cos b+ sin a sin b=cos (a-b))

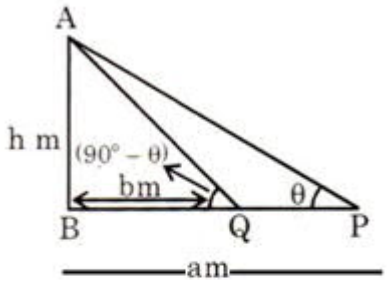
-[cos x cos (x-y) +sin x(sin (x-y)]

−cos [x-x+y] = -cos y

Option (1)

S15. Ans.(c)
Sol.

Let AB = h m



∠APB = θ°

From Δ ABP,

$\tan\theta = \frac{AB}{BP} = \frac{h}{a}$ .....(i)

From Δ ABQ,

$\tan(90° - \theta) = \frac{AB}{BQ}$

$\cot\theta = \frac{h}{b}$

$\tan\theta = \frac{b}{h}$ .....(ii)

From equation (i) and (ii),

$\frac{h}{a} = \frac{b}{h}$

$h^2 = ab$

$\therefore h = \sqrt{ab}$ m

AQ=$\sqrt{ab+b} = \sqrt{b(a+1)}$

S16. Ans.(d)

Sol.

Let A = B = C = D

So,

$\sin 0 \cos 0 + \sin 0 \cos 0 + \sin 0 \cos 0$

$= 0$

S17. Ans.(b)

Sol.

$$\Rightarrow \frac{4 \sin A.\cos^3 A - 4 \cos A \sin^3 A}{-\sin 4A}$$

$$\Rightarrow \frac{4 \sin A \cos A(\cos^2 A - \sin^2 A)}{-2 \sin 2A \cos 2A}$$

$$\Rightarrow \frac{4 \sin A \cos A \cos 2A}{-2 \sin 2A \cos 2A}$$

$\Rightarrow -1$ since $[\sin 2A = 2 \sin A \cos A]$

S18. Ans.(b)

Sol.

Pythagorean triplets, we know that (8, 15, 17) is

$\therefore\ 17\left(\frac{8}{17}\cos A + \frac{15}{17}\sin A\right)$

Let there be a angle B for which $\sin B = \frac{8}{17}$, $\cos B = \frac{15}{17}$

$= 17(\sin B \cos A + \cos B \sin A) + 15$

$= |17 \sin(A + B)| + 15$

We know that

$\sin(A + B)_{max} = 1$

$\sin(A + B)_{min} = -1$

$\therefore$ max value = $17 \times 1 + 15 = 32$

Min value = $17 \times (-1) + 15 = -2$

S19. Ans.(b)

Sol.

$\sin\frac{\theta}{2}\sin\frac{9\theta}{2} + \cos\frac{3\theta}{2}\cos\frac{13\theta}{2}$

$= \frac{1}{2}[2 \sin\frac{\theta}{2}\sin\frac{9\theta}{2} + 2\cos\frac{3\theta}{2}\cos\frac{13\theta}{2}]$

$\left[\begin{array}{c}\text{As we know } 2 \sin A \sin B = \cos(A-B) - \cos(A+B) \\ 2 \cos A \cos B = \cos(A+B) + \cos(A-B)\end{array}\right]$

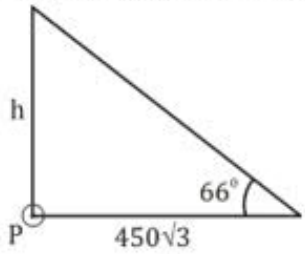
$\Rightarrow \frac{1}{2}[\cos 4\theta - \cos 5\theta + \cos 8\theta + \cos 5\theta]$

$\Rightarrow \frac{1}{2}[\cos 4\theta + \cos 8\theta]$

$= \frac{1}{2} \times 2 \cos 6\theta . \cos 2\theta$ since (cos A +cos b=2cos(a+b)/2 .cos(a-b)/2)

$= \cos 6\theta . \cos 2\theta$

S20. Ans.(b)
Sol.
Given below fig is drawn according to question



$\tan 60 = \frac{h}{450\sqrt{3}}$

$h = 450\sqrt{3} \times \sqrt{3}$

in 6 minutes, it achieved a height of 1350 m

$\text{speed} = \frac{450\times3}{6\times60} = 3.75 \text{ m/s}$

S21. Ans.(b)
Sol.

$$\frac{\text{Speed of Man}_1}{\text{Speed of Man}_2} = \sqrt{\frac{\text{Time taken by Man}_2}{\text{Time taken by Man}_1}}$$

$$\frac{8}{\text{Speed of Man}_2} = \sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}} = \sqrt{\frac{24}{5} \times \frac{3}{10}} = \sqrt{\frac{36}{25}}$$

$$\text{Speed of Man}_2 = \frac{5}{6} \times 8 = \frac{20}{3} \text{ km/hr}$$

$$= 6\frac{2}{3} \text{ km/hr}$$

S22. Ans. (b)
Sol.

Let slower speed = x

$x \times 4.5 \text{ hr} = (x + 5) \times (4.5 - 0.5) \text{ hr.}$

$x = 40 \text{ km/hr.}$

S23. Ans.(b)
Sol.

Speed of boat in still water = xkm/h (say)
and that of stream = y km/h
Then,

$x + y = \frac{1}{\frac{7.5}{60}} = \frac{1\times600}{75} = 8$

$x + y = 8 \text{ and } x - y = 5$

$\text{So, Speed of boat} = \frac{1}{2}(8 + 5)$

$= 6.5 \text{ km/h}$

S24. Ans.(b)
Sol.

Speed = 72 km/hr

T = 9 min

$D = S \times T$

$D = 72 \times \frac{9}{60} = \frac{54}{5}$ km

To get in 8 min

Speed should be $= \frac{54 \times 60}{5 \times 8} = 81$ km/hr

increased speed = (81 – 72) = 9 km/hr

S25. Ans.(a)
Sol.

Ist case

T_A	T_B
x+1	x

On engins failure for train B

	Old	New
S_B →	3	2
T_B →	2	3

×4 1 = (3+1) 4 hour

B takes 8 hour

So, A takes 9 hour

Speed of A $= \frac{720}{9} = 80$ km/hr

S26. Ans.(b)
Sol.

$S_1 = 60,\ S_2 = 108$

ACCORDING TO QUESTION

$\frac{D}{60} - \frac{D}{108} = 2$

$48D = 2 \times 60 \times 108$

D = 270 km

Alternate

S_A	:	S_B
60	:	108
5	:	9

→

T_A	:	T_B
9	:	5

×½ 4 = 2 hour, 1 = ½ hour

Distance = 9× ½ ×60=270

Or

Distance = 5 × ½ × 108 = 270

S27. Ans. (b)
Sol.

Let's 't' time taken to arise the water level by 7 cm.

$$\text{Now radius of Pipe} = \frac{14}{2} = 7 \text{ cm}$$

⇒ water flow by pipe = volume of tank

$$\pi \times \frac{7}{100} \times \frac{7}{100} \times 5 \times \frac{5}{18} \times t = 50 \times 44 \times \frac{7}{100}$$

$t = 7200$ sec

$$t = \frac{7200}{60 \times 60} = 2h$$

S28. Ans.(b)
Sol.

Suppose the distance b/w X & Y be K km.

Then, it takes $\frac{\frac{3}{4}K}{B}$ hours to cover

$\frac{3}{4}$ K km.

& It takes $\frac{\frac{1}{4}K}{S}$ hrs to cover $\frac{1}{4}$ K km

Average Speed

$$= \frac{\frac{3}{4}K+\frac{1}{4}K}{\frac{\frac{3}{4}K}{B}+\frac{\frac{1}{4}K}{S}} = \frac{K(BS)}{\frac{3}{4}KS+\frac{1}{4}KB}$$

$$= \frac{4BS}{3S+B} \text{ km/hr}$$

S29. Ans.(c)
Sol.

Distance is $12 \times \frac{9}{2} = 54$ km

New time = 3 hr

Therefore, new speed $= \frac{54}{3} = 18$ km/hr

S30. Ans.(d)
Sol.

ATQ,

Length are same

$(x - 6) \times 5 = (x - 7.5) \times 5.5$

$x = 22.5$ km/hr

And length = $(22.5 - 6) \times 5 \times 5/18$

= 22.92 metre