Solutions

S1. Ans.(a)

Sol. $\frac{4}{3} \times \pi \times 21 \times 21 \times 21 = \frac{1}{3} \times \pi \times \frac{21}{2} \times \frac{21}{2} \times h$

H = 336 cm

S2. Ans.(b)

Sol. Radius = 3.5

Distance travelled by the wheel in 20 revolutions

$= 2 \times \frac{22}{7} \times 3.5 \times 20$

= 440

S3. Ans.(b)

Sol.

$$= \left[\frac{1}{1 - x^{(p-q)}} + \frac{1}{1 - x^{(q-p)}}\right]$$

$$= \left[\frac{1}{1 - x^{(p-q)}} + \frac{1}{1 - x^{-(p-q)}}\right]$$

$$= \left[\frac{1}{1 - x^{(p-q)}} + \frac{1}{1 - \frac{1}{x^{(p-q)}}}\right]$$

$$= \left[\frac{1}{1 - x^{(p-q)}} + \frac{x^{(p-q)}}{x^{p-q} - 1}\right]$$

$$= \left[\frac{1}{1 - x^{(p-q)}} - \frac{x^{(p-q)}}{1 - x^{p-q}}\right]$$

$$= \left[\frac{1 - x^{(p-q)}}{1 - x^{(p-q)}}\right] = 1$$

S4. Ans.(a)

Sol. $x^2 - 3x + 1 = 0$

Multiply the equation by $\frac{1}{x}$

$$x - 3 + \frac{1}{x} = 0$$

$$x + \frac{1}{x} = 3$$

S5. Ans.(c)

Sol. $a + b + c = -11$

Or $(a + 4) + (b + 5) + (c + 2) = 0$

We know in $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx]$

If $x + y + z = 0$

Then, $x^3 + y^3 + z^3 - 3xyz = 0$

So, the value of given equation is 0.

S6. Ans.(c)

Sol. $\sqrt{7x + 12} + \sqrt{7x - 12} = 3 + \sqrt{33}$

By putting the value from options one by one, only

x = 3 satisfy the given equation.

S7. Ans.(a)

Sol. Perimeter = 44

$\therefore$ Side = $\frac{44}{4} = 11$

$\therefore$ Diagonal = $\sqrt{2} \times Side$

$= 11\sqrt{2}$

S8. Ans.(b)

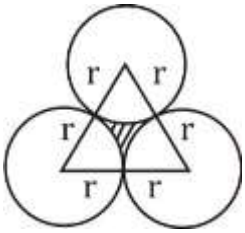
Sol. C.S.A = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 10$$

$= 880\ cm^2$

S9. Ans.(d)

Sol.



Area inclosed by three circles

$$= \left[\frac{\sqrt{3}}{4} \times (2r)^2\right] - \left[\frac{60}{360} \times \pi r^2 \times 3\right]$$

$$= \left[\sqrt{3}r^2\right] - \left[\frac{\pi}{2}r^2\right]$$

$$= \left[\sqrt{3} - \frac{\pi}{2}\right]r^2$$

$$= \left[\sqrt{3} - \frac{22}{7 \times 2}\right] \times 63 \times 63$$

$$= (3969\sqrt{3} - 6237)$$

S10. Ans.(a)

Sol. $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$

$\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{x}{y}\right)^{-(17-3a)} = \left(\frac{x}{y}\right)^{-17+3a}$

As base are same

$\therefore$ 5a – 3 = -17 + 3a

2a = – 14

a = – 7

S11. Ans.(a)

Sol. Put x = 0

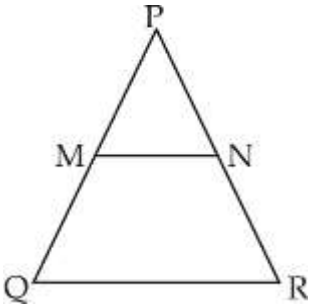
$\Rightarrow \frac{-6}{-12} \div \frac{6}{12}$

$\Rightarrow 1$

S12. Ans.(b)

Sol.

Area of □MNPQ = Area of ΔPQR – Area of ΔPMN.



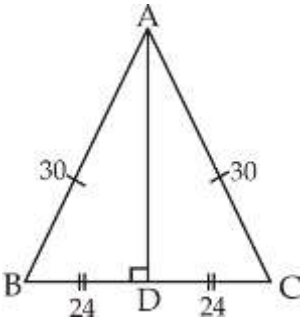
Area of ΔPQR = $9R^2 = 360$

Area of ΔPMN = $R^2 = 40$

Area of □MNPQ = 360 – 40 = 320

S13. Ans.(a)

Sol.



Note =In isosceles triangle median is also the altitude.

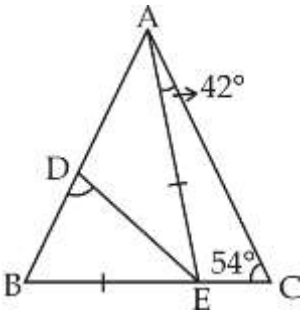
So, $AD^2 = AB^2 - BD^2$ (by Pythagoras theorem)

$= 30^2 - 24^2$

$AD = 18$

S14. Ans.(b)

Sol.



In ΔAEC ; $\angle AEC = 180° - (54 + 42)$

$= 84°$

By exterior angle property

$\angle BAE + \angle ABE = 84°$ - ...(i)

In ΔABE,

For perpendicular bisector, AE = BE

So, $\angle ABE = \angle BAE = x$

By Equation (i)

$2x = 84°$

$x = 42°$

S15. Ans.(c)

Sol. $\cos\theta + \sec\theta = 2$

If $\cos\theta = 1$ the $\sec\theta$ is also 1

Then, $\cos^{100}\theta + \sec^{100}\theta$.

$= 1 + 1 = 2$

S16. Ans.(c)

Sol. $1 + \cot A.\cot\left(\frac{A}{2}\right)$

Let $A = 60°$

Then, $1 + \sqrt{3} \times \frac{1}{\sqrt{3}} = 2$

Put $A = 60°$ in options, by option (c)

$\frac{1}{2} \times \text{cosec}^2(\frac{60°}{2}) = 2$

S17. Ans.(c)

Sol.

$\tan\frac{\theta}{2}.\tan\frac{2\theta}{5} = 1$

$\tan\frac{\theta}{2} = \cot\frac{2\theta}{5}$

$\tan\frac{\theta}{2} = \tan\left(90 - \frac{2\theta}{5}\right)$

$\frac{\theta}{2} = 90 - \frac{2\theta}{5}$

$\frac{\theta}{2} + \frac{2\theta}{5} = 90$

$\theta = 100°$

S18. Ans.(a)

Sol. $x - y = 7$

take

$x = 15$

& $y = 8$ then satisfied the eqn.

So, $(x - 15)^3 - (y - 8)^3 = (15 - 15)^3 - (8 - 8)^3$

$= 0$

S19. Ans.(d)

Sol. $x - y - \sqrt{18} = -1, x + y - 3\sqrt{2} = 1$

$x - y = \sqrt{18} - 1$ ....(i)

$x + y = 1 + 3\sqrt{2}$ ....(ii)

So,

$(x^2 - y^2) = 17$

From (i) & (ii)

$x = \sqrt{18}$

$y = 1$

So,

$12xy\,(x^2 - y^2) = 12 \times \sqrt{18} \times 1\,(17)$

$= 612\sqrt{2}$

S20. Ans.(b)

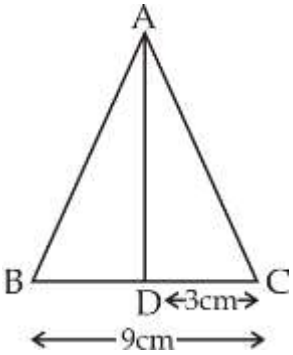
Sol.

$\frac{p}{q} = \frac{r}{s} = \frac{t}{4} = \sqrt{5}$

then, $\left(\frac{3P^2+4r^2+5t^2}{3q^2+4r^2+5u^2}\right) = \left(\sqrt{5}\right)^2 = 5$

S21. Ans.(b)

Sol.



BD = 6 cm

DC = 3 cm

Height will be the same of both triangles

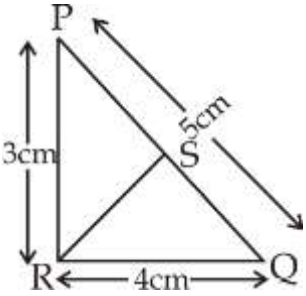
So,

Area of $\Delta = \frac{1}{2} \times$ base $\times$ height

$\frac{\text{Area of ABD}}{\text{Area of ADC}} = \frac{BD}{DC} = \frac{6}{3} = \frac{2}{1}$

S22. Ans.(a)

Sol.



PR = 3 cm

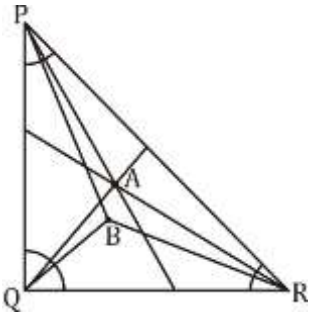
RQ = 4 cm

PQ = 5 cm

$RS = \frac{PR \times RQ}{PQ} = \frac{3 \times 4}{5} = \frac{12}{5}$

S23. Ans.(c)

Sol.



Given that, ∠PBR = 105°

we know that.

⇒ Point of intercession of angle bisector = Incentre

⇒ Point of intercestion of Altitude = orthocenter.

So, $\angle PBR = 90° + \frac{\angle Q}{2} = 105°$ (Incenter property)

And ∠PAR + ∠PQR = 180° (orthocenter property)

∠PAR = 180° – 30° = 150°

S24. Ans.(d)

Sol. If we draw 4 lines, then the no. of possibility by using options.

(a)  ⇒ no intersecting points

(b)  ⇒ 5 intersecting points

(c)  ⇒ 4 intersecting points

So, only remaining option (d)

S25. Ans.(b)

Sol. 1200 is multiplied by 3 then it becomes a perfect square.

1200 × 3 = 3600

3600 is square root of 60.

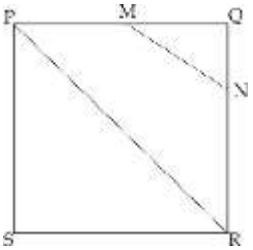
Alternately,

1200 = 2 × 2 × 3 × 10 × 10

Thus, if 3 is multiplied, then the number would be perfect square.

S26. Ans.(d)

Sol. PQRS is a square



Let side of square PQRS = a cm

$\therefore PM = MQ = \frac{a}{2}$ cm

$NR = \frac{2}{3}QR = \frac{2}{3}a$ cm

$QN = a - \frac{2}{3}a = \frac{a}{3}$ cm

Area of ΔMNQ

$\frac{1}{2} \times \frac{a}{2} \times \frac{a}{3} = 48$

$a^2 = 576$ cm^2

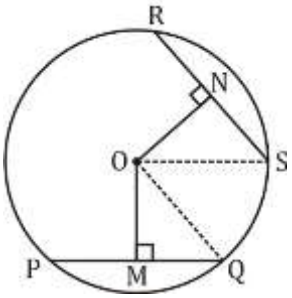
$\therefore a = 24$ cm

And since PR is diagonal of square PQRS

$\therefore PR = \sqrt{2}a = 24\sqrt{2}$ cm

S27. Ans.(c)

Sol.



We know that perpendicular drawn from centre to any chord divides the chord in two equal parts.

$\therefore PM = MQ = 15$ cm

$RN = NS = 12$ cm

OQ is radius of circle

$OQ = \sqrt{(MO)^2 + (MQ)^2}$

$= \sqrt{(12)^2 + (15)^2}$

$= \sqrt{369}$

$OQ = OS$

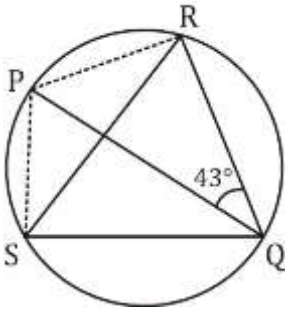
$\therefore ON = \sqrt{(OS)^2 - (NS)^2}$

$= \sqrt{369 - 144}$

$= \sqrt{225} = 15$ cm

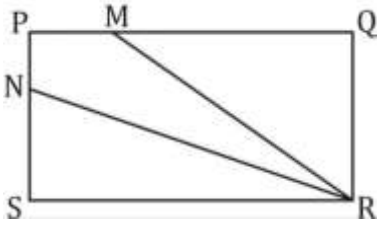
S28. Ans.(c)

Sol.



Joining P to R and P to S
We know angle subtended by same chord at the circle subtends equal angle.
∴ ∠PSR = ∠PQR = 43°
And also angle subtended by diameter at the circle is 90°
∴ ∠PSQ = 90°
∴ ∠ QSR = 90 – 43 = 47°

S29. Ans.(b)
Sol.



Let PS = QR = b cm
RS = QP = ℓ cm
Area of PMRN
= area of PQRS – [area of ΔMQR + area of ΔNRS]
$= \ell \times b - \left[\frac{1}{2}\left(\frac{2}{3}b \times \ell\right) + \frac{1}{2}\left(\frac{2}{3}\ell \times b\right)\right]$
$= \ell b - \frac{2}{3}\ell b = \frac{1}{3}\ell b = 17 cm^2$
Area of PQRS = $\ell b = 51\ cm^2$

S30. Ans.(b)
Sol. (1 – sin A cos A) (sin A + cos A)
= sin A – $\sin^2$ A cos A + cos A – sin A $\cos^2$ A
= sin A – sin A $\cos^2$ A + cos A – $\sin^2$ A cos A
= sin A [1 – $\cos^2$ A] + cos A [1 – $\sin^2$ A]
= $\sin^3$ A + $\cos^3$ A [∵ $\sin^2$ A + $\cos^2$ A = 1].

S31. Ans.(a)
Sol. Let the fraction be x
ATQ,
$4x + \frac{6}{x} = 11$
$4x^2$ - 11x + 6 = 0
$4x^2$ - 8x – 3x + 6 = 0
4x(x – 2) – 3 (x – 2) = 0
(4x – 3) (x – 2) = 0
$x = \frac{3}{4}$ (as it is given in option)

S32. Ans.(c)

Sol. $T_3 = -13$

$a + 2d = -13$ ...(i)

$T_6 = -4$

$a + 5d = -4$ ...(ii)

on solving (i) and (ii) we get,

$a = -19$, & $d = 3$

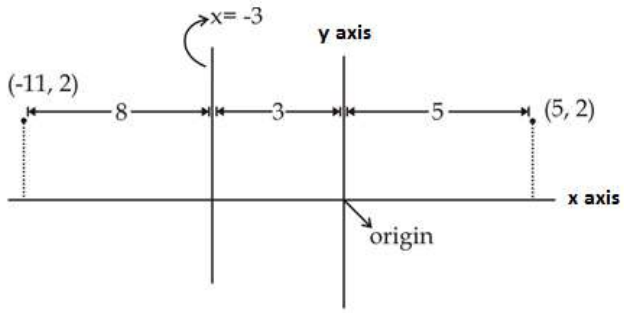
sum of 12 terms $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2 \times (-19) + (12-1) \times 3]$

$= -30$

S33. Ans.(a)

Sol.



S34. Ans.(a)

Sol.

Centroid $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{1 + 4 + (-2)}{3}, \frac{-5 + 0 + 2}{3}\right)$

$= (1, -1)$

S35. Ans.(d)

Sol. ATQ,

$\frac{3 - (-5)}{-5 - x} = \frac{-4}{3}$

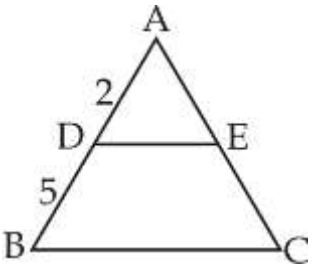
$8 \times 3 = -4(-5 - x)$

$24 = 20 + 4x$

$x = 1$

S36. Ans.(a)

Sol.



∵ ΔADE ≈ ΔABC

$\frac{Ar(\Delta ADE)}{Ar\ (\Delta ABC)} = \left(\frac{AD}{AB}\right)^2$

$\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \left(\frac{2}{2+5}\right)^2 = \frac{4}{49}$

$\frac{Ar(\Delta ADE)}{Ar(\square BDEC)} = \frac{4}{49-4} = 4:45$ {As, $Ar(\square BDEC) = Ar\ (\Delta ABC) - Ar(\Delta ADE)$ }

S37. Ans.(d)

Sol. x = $\left(\sqrt{5}+\sqrt{2}\right)^2$

$\sqrt{x} = \sqrt{5}+\sqrt{2}$

$\frac{1}{\sqrt{x}} = \frac{\sqrt{5}-\sqrt{2}}{3}$

$= \frac{1}{3}\left(\sqrt{5}-\sqrt{2}\right)$

$\sqrt{x}-\frac{1}{\sqrt{x}} = \left(\sqrt{5}+\sqrt{2}\right)-\frac{1}{3}\left(\sqrt{5}-\sqrt{2}\right)$

$= \frac{2}{3}\sqrt{5}+\frac{4}{3}\sqrt{2} = \frac{2}{3}\left(\sqrt{5}+2\sqrt{2}\right)$

S38. Ans.(b)

Sol. $\sqrt{7}+\sqrt{3}$&$\sqrt{5}+\sqrt{5}$ *and* $\sqrt{2}+\sqrt{8}$

Squaring all terms

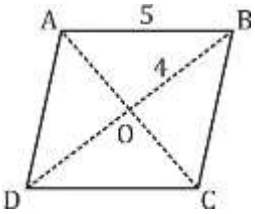
$10+2\sqrt{21}$, $10+2\sqrt{25}$, $10+2\sqrt{16}$

Hence

$\sqrt{5}+\sqrt{5} > \sqrt{7}+\sqrt{3} > \sqrt{2}+\sqrt{8}$

S39. Ans.(b)

Sol.

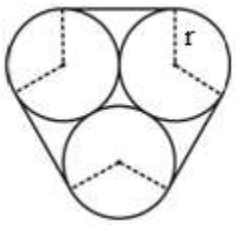


∵ diagonal of rhombus intersect at right angle hence

AO = 3 cm ⇒ AC = 6 cm

Area of rhombus = ½ ×6 × 8 = 24 cm

S40. Ans.(c)
Sol.



Curved string = $2\pi r = 18\pi$ cm
Straight string = $3 \times (28) = 54$ cm
Total length of string = $54 + 18\pi$ cm

S41. Ans.(b)
Sol. $(2\pi - 1)\, r = 111$

$$\left(\frac{44}{7} - 1\right) r = 111$$

$$\frac{37}{7} r = 111$$

$r = 21$ cm

$$\text{Area of circle} = \frac{22}{7} \times 21 \times 21$$

$$= 1386 \text{ cm}^2$$

S42. Ans.(a)
Sol. $d = 14$ cm
$r = 7$ cm
C.S.A. of a sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7$$

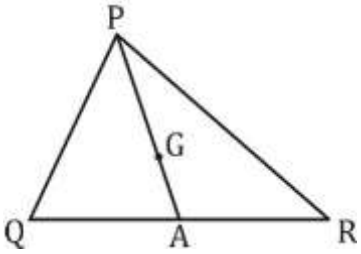
$$= 616 \text{ cm}^2$$

S43. Ans.(d)
Sol.

$$\frac{V_1}{V_2} = \frac{a_1^3}{a_2^3}$$

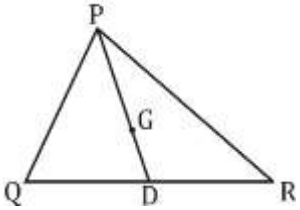
$$\left(\frac{11}{13}\right)^{\frac{1}{3}} = \frac{a_1}{a_2}$$

S44. Ans.(a)
Sol.



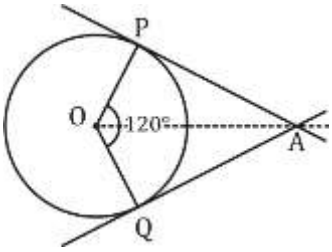
Let PA = 3
Then PG = 2 (∵ G is centroid hence PG:GA= 2:1)
GA = 1
Then ,
= (PA + GA) : (PG – GA)
= (3 + 1) : (2 – 1)
= 4 : 1

S45. Ans.(b)
Sol.



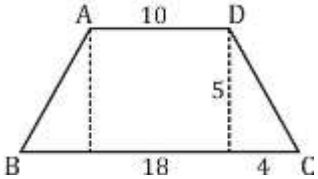
If PG = 24 cm
Then, GD = 12 cm
PD = 36 cm

S46. Ans.(c)
Sol.



∠PAQ= 120/2= 60^0
∠PAO = 30^0
∠PAQ : ∠PAO = 2 : 1

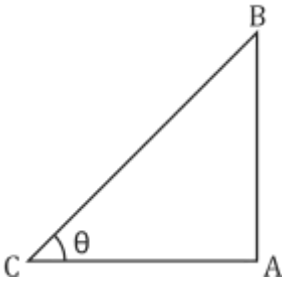
S47. Ans.(c)
Sol.



Area of trapezium = ½ (sum of parallel sides) × height
70 = ½ (10 + 18) × height
140 = 28 × height
Height = 5 cm
CD = $\sqrt{25 + 16}$ =$\sqrt{41}$ cm

S48. Ans.(a)

Sol. Let AB be the tree and AC be its shadow.



Let ∠ACB = θ,

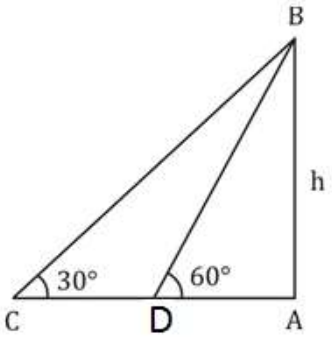
Then

$\frac{AC}{AB} = \sqrt{3} \Rightarrow \cot\theta = \sqrt{3}$

∴ θ = 30°

S49. Ans.(d)

Sol.

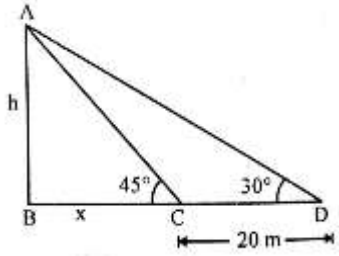


One of AB, AD and CD must have given.

So, the data is inadequate

S50. Ans.(c)

Sol.



In Δ ABC, $\tan 45° = \frac{AB}{BC} = \frac{h}{x}$

$1 = \frac{h}{x}$

h = x .....(i)

$\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+20}$

$x + 20 = \sqrt{3}h$

$h + 20 = \sqrt{3}h$

$20 = (\sqrt{3} - 1)h$

$h = \frac{20}{\sqrt{3}-1}$

$= \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{20\,(\sqrt{3}+1)}{2} = 10\,(\sqrt{3}+1)\,m$

Hence the height is $10\,(\sqrt{3}+1)\,m$