Quantitative Aptitude

Geometry (Circle, Chord and Tangent)

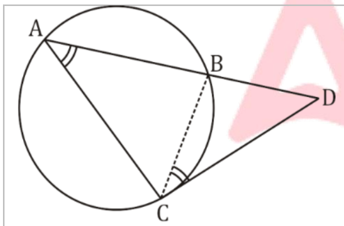
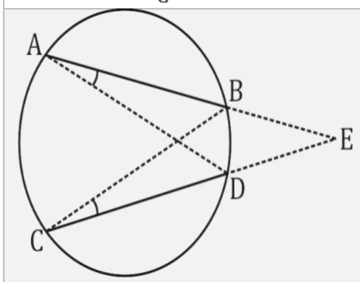
Circle – Basic Definitions and Important Properties

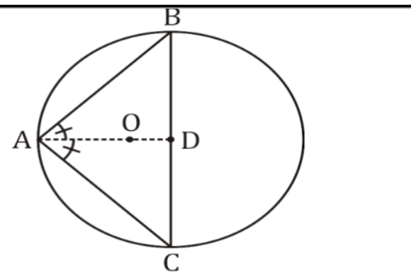
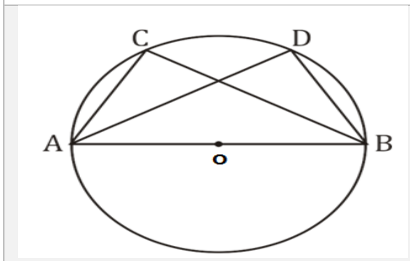
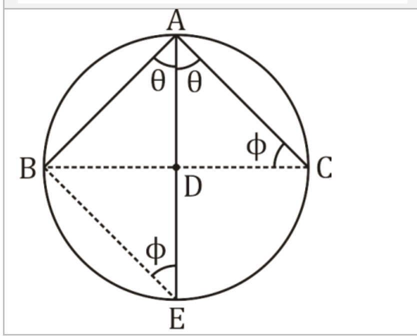
Category	Definition / Property
Circle	Set of points equidistant from a fixed point (centre).
Radius	Distance from the centre to any point on the circle.
Chord	Line segment joining two points on the circle.
Centre	Fixed point from which all points on the circle are equidistant.
Circumference	Outer boundary of the circle.

Important Properties of Circle

S.No.	Property
1.	Equal chords subtend equal angles at the centre and vice versa.
2.	Equal chords are equidistant from the centre and vice versa.
3.	The perpendicular from the centre to a chord bisects the chord.
4.	The angle subtended by a chord at the centre is twice the angle it subtends at any point on the major arc.
5.	Angle made by a diameter on the circle (circumference) is always 90°.
6.	If two chords intersect inside a circle, the product of the segments of one chord is equal to the product of the other.

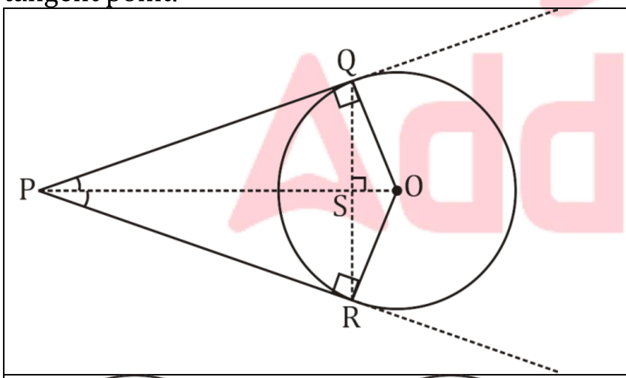
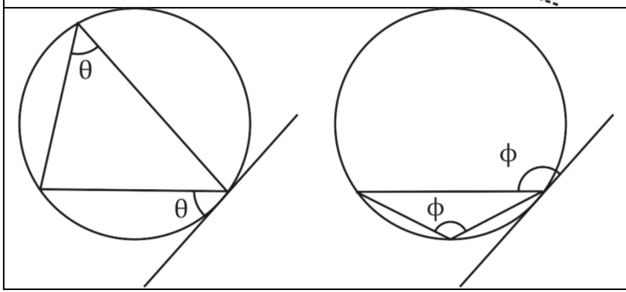
Important Theorems of circle:

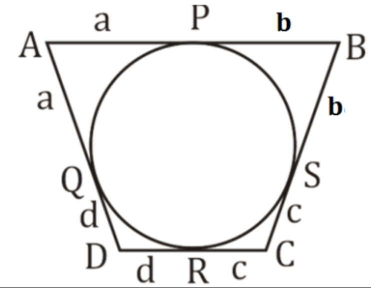
	If a secant and a tangent externally intersect each other. Then $DC^2 = DA \times DB$
	If two chords intersect outside a circle or two secant intersect each other, then $EA \times EB = EC \times ED$

	Two chords AB and AC of a circle are equal. Then the centre of the circle lies on the angle bisector of ∠BAC.
	Angles by the same segment of a circle are equal. i.e., ∠ACB = ∠ADB
	In the above figure, AE is angle bisector of ∠BAC then $AB.AC + DE.AE = AE^2$

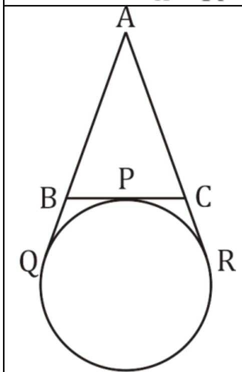
Tangents and Its Properties:

A line segment touches only at one point of a circle is called a tangent of the circle and the point of contact or tangent point.

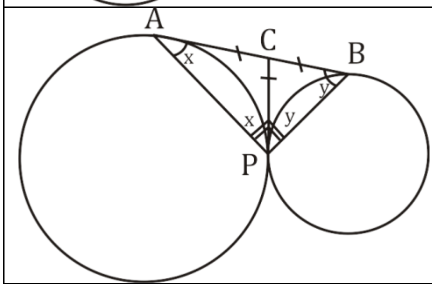
	1. The line joining centre of circle and point of contact is perpendicular to the point. 2. Tangents drawn from an external point to a circle (a) from a point outside a circle maximum two tangents can be drawn and both the tangents are equal PQ = PR (b) ∠QPO = ∠OPR = ∠SQO (c) ∠QOP = ∠POR = ∠PQS (c) PQQO=QSSO
	**Alternate segment Theorem** The angle between a chord and a tangent drawn at end point of chord is equal respectively to the angle formed in the corresponding alternate segments.



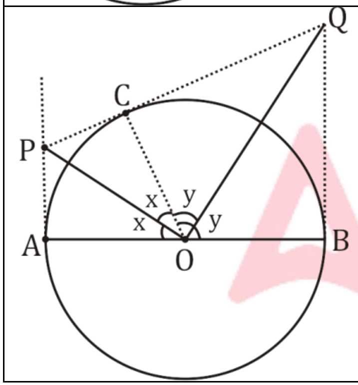
If a quadrilateral circumscribed a circle.
Then sum of opposite are equal
AB+DC=AD+BC



A circle externally touch side BC of a ΔABC at P, AB produced at Q and AC produced at R. if AQ = a cm, then what is perimeter of ΔABC?
Perimeter = 2 × AQ = 2a

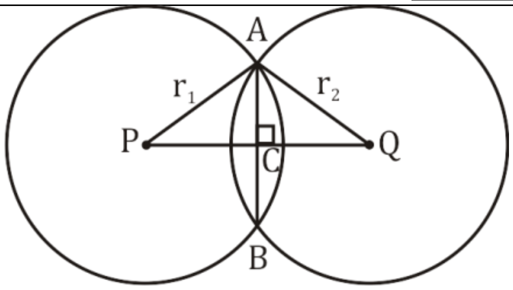


Two cirles externally touch each other at P. AB is direct common tangent (DCT) of the circles. If ∠BAP = x then find ∠ABP = ?
Remember ∠APB is always right angle
∠ABP = 90 – x

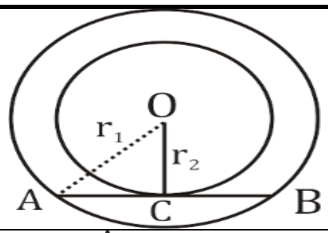
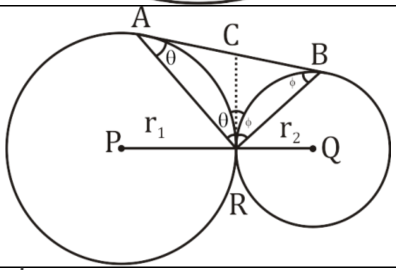
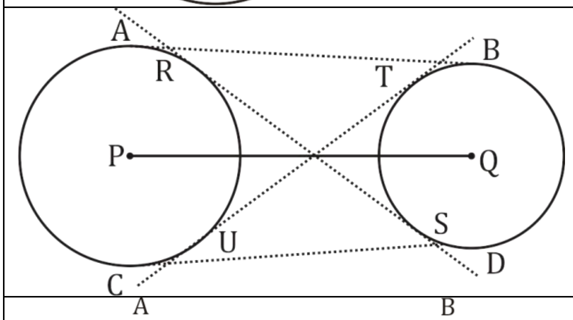


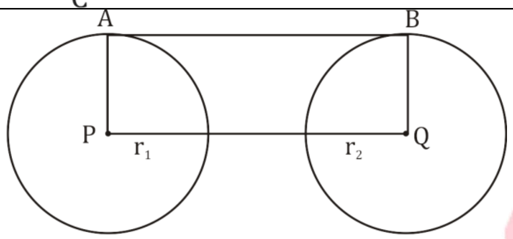
AB is a diameter of a circle. Two tangents drawn at A & B. Tangent drawn at any point C of the circle meet both tangents
at P & Q. Find angle ∠POQ = ?
Remember ∠POQ = 90°

Two Circles related Concepts:



If circles intersect each other, AB is common chord of the circles
Distance between centres PQ = PC + CQ
=r12-AB22+r22-AB22
Case 1:
If AQ is tangent of circle 1 or PA is tangent of circle 2 then ΔPAQ is a right angled triangle and AC⊥PQ
Then PA×AQ=PQ×AC or AC=PA×AQPQ
Length of common chord AB =2AC⇒2PA×AQPQ
Case 2: If each circles passes through the centre of the other then length of common chord is 3r.

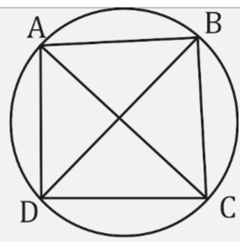
<table>
<tr><td>
</td><td>If circles are concentric
AB is chord of the greater circle be a tangent to the smaller circle length of AB = 2AC =2r12-r22</td></tr>
<tr><td>
</td><td>If circles externally touch each other
AB is common chord of the circles
Distance between centre = $r_1 + r_2$
R is common point of circles
AB is direct common tangent (DCT)
Then ∠ARB = 90°</td></tr>
<tr><td>
</td><td>If circles placed at some distance

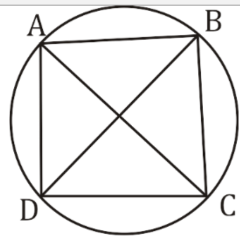
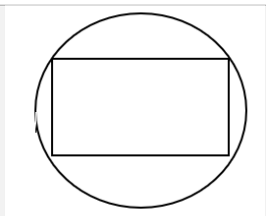
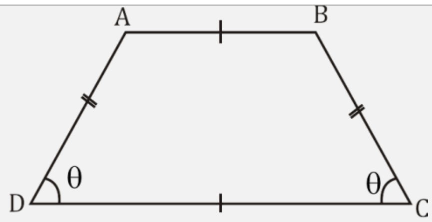
Direct common tangent (DCT)
AB and CD =Distance between centres2-r1-r22
Transverse common tangent (TCT)
RS and TU =Distance between centres2-(r1+r2)^2</td></tr>
<tr><td>
</td><td>Case 1
If radius of circles are same r1=r2=r
DCT =PQ2-r-r
AB=PQ2
AB = PQ i.e. length of direct common tangent equal to distance between circle
CASE 2:
If circle externally touch each other
i.e. distance between centre = $r_1 + r_2$</td></tr>
</table>

Number of Common Tangents Between Circles:

Condition	**Max. No. of Common Tangents**
Circles intersect each other	2
Circles touch each other	3 (External), 1 (Internal)
Circles neither touch nor intersect	4 (External), 0 (Internal)

Cyclic Quadrilateral:

<table>
<tr><th>S.No.</th><th>Theorem / Property / Equation</th><th>Description / Explanation</th><th>Figure Description</th></tr>
<tr><td>1.</td><td>Definition: A quadrilateral is cyclic if all its vertices lie on a circle.</td><td>The circle is called the circumcircle of the quadrilateral.</td><td>
</td></tr>
<tr><td>2.</td><td>∠A + ∠C = ∠B + ∠D = 180°</td><td>Opposite angles of a cyclic quadrilateral are supplementary.</td><td></td></tr>
</table>

3.	**Exterior Angle Theorem**: ∠Exterior = ∠Interior Opposite	The **exterior angle** at any vertex equals the **interior opposite angle**.	∠EAB = ∠C (E is outside circle on extended AB)
4.	**Ptolemy's Theorem**: AC × BD = AB × CD + AD × BC	In a cyclic quadrilateral, product of diagonals = sum of products of opposite sides.	
5.	**Right Angles in Cyclic Parallelogram**: All angles = 90°	A cyclic quadrilateral that is a **parallelogram** must be a **rectangle** (∵ ∠s = 90°).	
6.	**Isosceles Trapezium is Cyclic**	If non-parallel sides of a trapezium are equal, then it is a cyclic quadrilateral.	Trapezium ABCD with AB ∥ CD and AD = BC.
7.	**In Isosceles Trapezium**: ∠A = 180° – θ and ∠B = 180° – θ If sum of opposite angles is 180° (∠A + ∠C = ∠B + ∠D = 180°) Then it's cyclic quadrilateral	Since AB ∥ DC and ∠D = ∠C = θ, then adjacent angles on same side = supplementary.	
9.	**Equal Angles in Semicircle**: ∠ in semicircle = 90°	A special cyclic quadrilateral formed when diameter is one of the sides.	Triangle in semicircle extended to 4-sided figure.
10.	**Angle Sum Property**: ∠A + ∠B + ∠C + ∠D = 360°	Angle sum of any quadrilateral = 360°. For cyclic, opposite pairs add to 180°.	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