Quantitative Aptitude

Geometry (Triangle, Congruency & Similarity)

Category	Theorem Name	Statement	Conditions / Criteria	Applications
Congruency	**SSS (Side–Side–Side)**	Two triangles are congruent if their three sides are respectively equal.	All three sides equal: AB = PQ, BC = QR, CA = RP	Used when side lengths are given.
	SAS (Side–Angle–Side)	Two triangles are congruent if two sides and the included angle are equal.	AB = PQ, ∠B = ∠Q, BC = QR	Helpful when angle between two sides is known.
	ASA (Angle–Side–Angle)	If two angles and the included side of one triangle are equal to those of another triangle.	∠A = ∠P, AB = PQ, ∠B = ∠Q	Angle between two sides is known.
	AAS (Angle–Angle–Side)	Two angles and a non-included side equal implies congruency.	∠A = ∠P, ∠B = ∠Q, AC = PR	Common when angle and side data are not in sequence.
	RHS (Right angle–Hypotenuse–Side)	Right-angled triangles are congruent if hypotenuse and one side are equal.	∠B = ∠Q = 90°, Hypotenuse and one leg equal	Used in right triangles only.
	CPCT (Corresponding Parts of Congruent Triangles)	If triangles are congruent, then all their corresponding parts (angles and sides) are equal.	Applies **after proving congruency**	Used to deduce unknown sides/angles.
Similarity	**AAA (Angle–Angle–Angle)**	If two triangles have their angles equal, then they are similar.	∠A = ∠P, ∠B = ∠Q, ∠C = ∠R	Most common method for proving similarity.
	AA (Angle–Angle)	If two angles of one triangle are equal to two of another, triangles are similar.	∠A = ∠P, ∠B = ∠Q (3rd angle automatically equal)	Shortcut form of AAA similarity.
	SSS (Side–Side–Side) Similarity	If all three sides of two triangles are in the same ratio, then triangles are similar.	AB/PQ = BC/QR = CA/RP	Used when all three sides are given in ratio.
	SAS (Side–Angle–Side) Similarity	Two sides in same ratio and included angle equal → triangles are similar.	AB/PQ = AC/PR and ∠A = ∠P	Similar to SAS congruency but with ratio.
	Basic Proportionality Theorem (Thales)	A line parallel to one side of a triangle divides the other two sides in the same ratio.	DE ∥ BC ⇒ AD/DB = AE/EC	Often used in coordinate and diagram questions.
	Converse of BPT	If a line divides two sides of triangle in the same ratio, then it is parallel to the third side.	AD/DB = AE/EC ⇒ DE ∥ BC	Used in proving lines are parallel.
	Angle Bisector Theorem	An angle bisector in a triangle divides the opposite side in the ratio of the adjacent sides.	If AD is bisector of ∠A in △ABC, then BD/DC = AB/AC	Used to calculate unknown segments using ratio.

	Converse of Angle Bisector Theorem	If a point divides the opposite side in the ratio of adjacent sides, then it lies on angle bisector.	BD/DC = AB/AC ⇒ AD is angle bisector	Helpful in reverse reasoning.
	Area Ratio Theorem	If two triangles are similar, the ratio of their areas is equal to the square of the ratio of sides.	△ABC ~ △DEF ⇒ Area(ABC)/Area(DEF) = $(AB/DE)^2$	Used for comparing areas.
	Altitude/Median Ratio in Similar Triangles	Ratio of corresponding altitudes, medians, and angle bisectors = ratio of corresponding sides.	If △ABC ~ △DEF, then AD/DP = AB/DE, etc.	Helps in height-related calculations.

Triangle

Theorem / Concept	**Formula / Rule**	**Conditions / Triangle Type**	**Usage / Tip**
Angle Sum Property	∠A + ∠B + ∠C = 180°	All triangles	Always true, used for missing angle problems.
Exterior Angle Theorem	Exterior angle = ∠opposite interior 1 + ∠opposite interior 2	All triangles	Very useful in MCQ and diagram problems.
Triangle Inequality Theorem	Sum of any two sides > third side	All triangles	Use to check valid triangle formation.
Pythagoras Theorem	$Hyp^2 = Base^2 + Perpendicular^2$	Only right-angled triangle	Most used for length-related problems.
Converse of Pythagoras	If $a^2 + b^2 = c^2$ ⇒ triangle is right-angled at included angle	If side lengths are known	Helpful for triangle classification.
Area (Basic Formula)	Area = ½ × base × height	All triangles	Use for direct area calculation.
Area (Heron's Formula)	$\sqrt{[s(s-a)(s-b)(s-c)]}$, s = (a+b+c)/2	When all three sides are given	For side-based area questions.
Area Using Sine Rule	Area = ½ × ab × sin C	When two sides and included angle are known	Useful when angle is given instead of height.
Inradius (r) Formula	Area = r × s → r = Area/s	All triangles	For incenter/inradius-based problems.
Circumradius (R) in Right Triangle	R = Hypotenuse / 2	Only right-angled triangle	Simple and fast shortcut.
Circumradius (General)	R = (abc) / (4 × Area)	Any triangle	Use with Heron's or sine area.
Sum of Sides and Angles Rule	a / sin A = b / sin B = c / sin C = 2R	Law of Sines – all triangles	Ratio problems & unknown side/angle finding.
Cosine Rule (Generalized Pythagoras)	$a^2 = b^2 + c^2 - 2bc \cdot \cos A$	For non-right triangles	For unknown sides/angles when not right-angled.
Sine Rule (Law of Sines)	a / sin A = b / sin B = c / sin C	Any triangle	Often used with circumradius questions.

Area Ratio of Similar Triangles	$Area_1 / Area_2 = (side_1 / side_2)^2$	Only for similar triangles	Use when triangles are similar.
Midpoint Theorem	Line joining midpoints = ½ × base and ∥ base	All triangles	Shortcut for coordinate geometry-based triangle problems.
Angle Bisector Theorem	BD/DC = AB/AC	Angle bisector divides side in ratio of adjacent sides	Very important for ratio-based triangle division.
Apollonius Theorem	$AB^2 + AC^2 = 2AD^2 + ½ BC^2$	When D is midpoint of BC in triangle ABC	For median-related problems.
Equilateral Triangle Area	$(\sqrt{3} / 4) \times a^2$	Only equilateral triangles	Fast shortcut when all sides equal.
Number of Triangles in a Polygon (n sides)	No. of triangles = n – 2	Convex polygon	Useful in polygon-based MCQs.

Geometry (Properties and Theorems of Triangle)

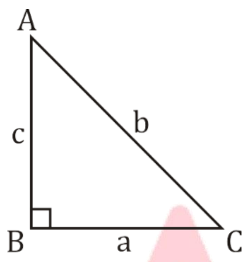
BASIC PROPERTIES OF TRIANGLES

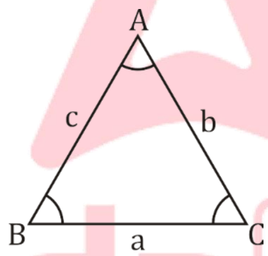
Property #1

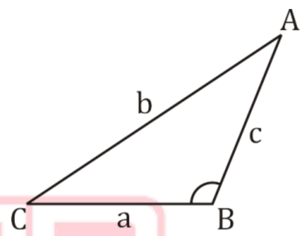
Right angled triangle
Largest side, $b^2 = a^2 + c^2$

Acute-angled triangle
Largest side, $b^2 < a^2 + c^2$

Obtuse-angled triangle
Largest side $b^2 > a^2 + c^2$

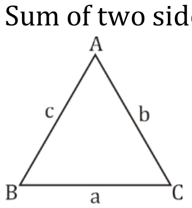






Property #2

Sum of two sides should be greater than the third side



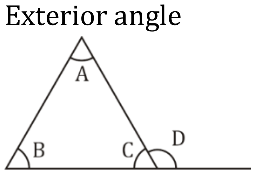
$a + b > c$, $b + c > a$, $c + a > b$

Difference of two sides should be smaller than third side

$|b - c| < a$, $|c - a| < b$, $|a - b| < c$

Property #3

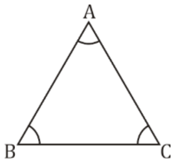
Exterior angle



∠D = exterior angle = ∠A + ∠B

Property #4

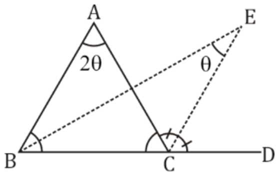
In ΔABC, if AB = AC, then ∠B = ∠C



Property #5

The angle between internal bisector of a base angle and external bisector of the other base angle is half of the remaining vertex angle.

i.e 2∠BEC=∠BAC.



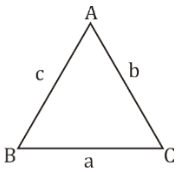
Property #6

⇒ Perimeter of triangle (p) = a + b + c

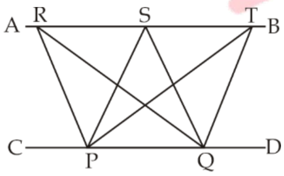
⇒ Semi-perimeter of triangle (s) = $\frac{p}{2}$ = a + b + c

<u>⇒ Area of triangle Δ</u>

1. Heron's formula, Δ = $\sqrt{s(s-a)(s-b)(s-c)}$



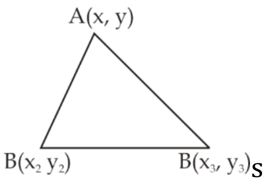
2. Δ = $\frac{1}{2} \times base \times height$
3. Δ = $\frac{1}{2}$ × ac × sin B = $\frac{1}{2}$ ab sin C = $\frac{1}{2}$ bc sin A
4. The area of triangles between two parallel line with same base are equal.



If AB||CD then

Ar. ΔPQR = Ar. ΔPQS = Ar. ΔPQT

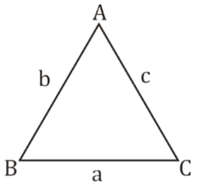
5. If coordinate of three vertex are given



$$\Delta = \frac{1}{2}|x_1(y_1 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Property #7

⇒ Sine formula



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

⇒Cosine formula

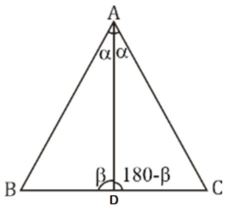
$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Property #8

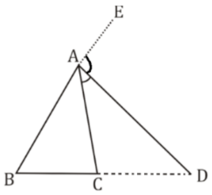
⇒ Interior Angle Bisector Theorem



If AD is angle bisector of ∠A, then $\frac{AB}{AC} = \frac{BD}{CD}$

⇒ Exterior Angle Bisector Theorem

AD is angle bisector of ∠CAE, then $\frac{AB}{AC} = \frac{BD}{CD}$



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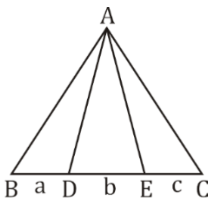
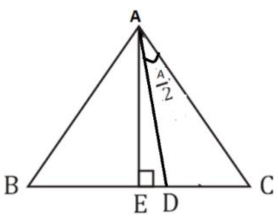


Area will also be divided in the ratio of a : b : c, because sides are in the ratio a : b : c

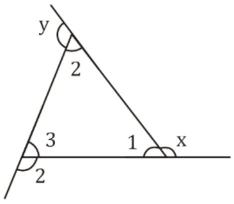
Property #9

In ΔABC, AE ⊥ BD and AD is angle bisector of ∠A, then $\angle EAD = \frac{1}{2}|\angle B - \angle C|$



Property #10

Sum of interior angles of a triangle is 180° and sum of exterior angles is 360°.

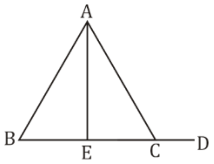


$\angle 1 + \angle 2 + \angle 3 = 180°$, $\angle x + \angle y + \angle z = 360°$

Property #11

In ΔABC, the side BC produced to D and angle bisector of ∠A meets BC at E, then

$\angle ABC + \angle ACD = 2\ \angle AEC$



Property #12

Total number of triangles with integral sides and perimeter n

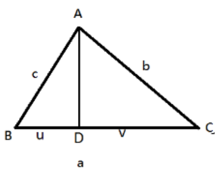
When n is odd : nearest integer $= \frac{(n+3)^2}{48}$

When n is even : nearest integer $= \frac{n^2}{48}$

Property#13

<u>Stewart's theorem</u>

In triangle ABC,



$AD^2 = \frac{ub^2+vc^2}{u+v} - uv$

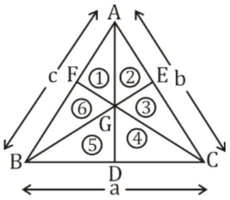
<u>CENTRES OF TRIANGLES</u>

<u>I. CENTROID</u>

It is the intersection point of all medians.

Median is the line joining a vertex and the mid-point of side opposite to the vertex.

Centroid (G) is same as the center of mass of a triangle.



1. Median divides the area of triangle into two equal area of triangles
   Area of ΔABD = Area of ΔACD
2. Area of six smaller triangles formed by 3 medians and 3 sides are equal and is equal to □ × Area ΔABC
3. Centroid G divides each median in the ratio 2 : 1.
   AG : GD = BG : GE = CG : GF = 2 : 1

4. Lengths of medians

AD = $\frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

BE = $\frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$

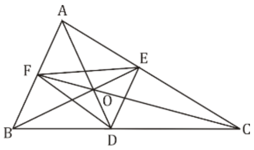
CF = $\frac{1}{2}\sqrt{2b^2 + 2a^2 - c^2}$

5. In a triangle three times of sum of square of sides equal to four times of sum of square of medians i.e $AB^2 + BC^2 + AC^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2)$

6. In a triangle, the ratio of the sum of sides to the sum of three medians is always greater than 2/3

$\frac{(AB+BC+AC)}{(AD+BE+CF)} > \frac{2}{3}$

7. Area of triangle formed by joining mid-points of two sides and centroid is $\frac{1}{12}$th of area of triangle.



Ar ΔOFE = Ar ΔOFD = Ar ΔOED = $\frac{1}{12}$ Ar ΔABC

O is also centroid of ΔDEF

8. Let m_1, m_2 & m_3 are three medians, then by Heron's formula.

Area of Δ = $\frac{4}{3}\sqrt{S_m(S_m - m_1)(S_m - m_2)(S_m - m_3)}$

Where, S_m = semi median = $\frac{m_1+m_2+m_3}{2}$

Note: If $m_1^2 + m_2^2 = m_3^2$

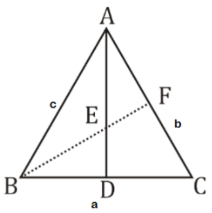
i.e. m_1, m_2 & m_3 forms Pythagorean triplet

then Area of Δ = $\frac{2}{3} m_1 m_2$

9. The line segment joining the mid-point of a median to vertex divides opposite side in the ratio 1 : 2.

⇒ E is mid-point of median AD, then

AF : FC = 1 : 2



10. The median from sides of length b and c are perpendicular if

$b^2 + c^2 = 5a^2$

If medians form Pythagorean triplets

i.e. $m_2{}^2 + m_3{}^2 = m_1{}^2$

then also result will be same.

$b^2 + c^2 = 5a^2$

If two medians are perpendicular then all medians will form Pythagorean triplets.

11. The sum of any two sides of a triangle is greater than twice the medians drawn to third sides

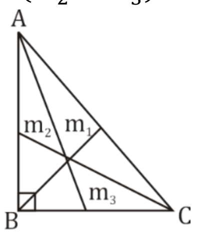
AB + AC > 2AD, AB + BC > 2BE, AC + BC > 2CF

→ Adding all ⇒ AB + AC + BC > AD + BE + CF

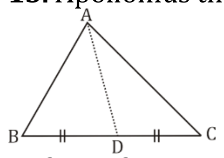
→ Sum of sides (Perimeter) is always greater than sum of all median.

12. In a right-angled triangle four times of sum of square of two medians (not right-angled vertex median) is equal to five times of square of hypotenuse.

$4(m_2^2 + m_3^2) = 5AC^2$



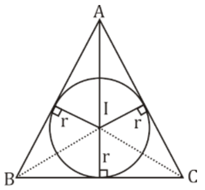
13. Apollonius theorem



$AB^2 + AC^2 = 2(AD^2 + BD^2)$

II. INCENTRE

→ It is the intersection point of angle bisector of a triangle.



1. The length of perpendicular drawn from the incentre to the all the three sides are equal and it is called inradius of the triangle.
2. The angle between line segment joining the incentre and two vertex is equal to sum of the half of third vertex angle and right angle.

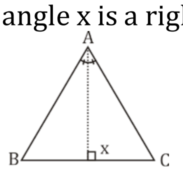
$\angle BIC = 90 + \frac{\angle A}{2}$,

$\angle AIC = 90° + \frac{\angle B}{2}$,

$\angle BIA = 90° + \frac{\angle C}{2}$

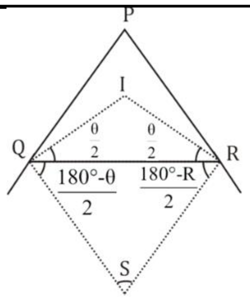
3. Incentre is the only center which is equi-perpendicular distance from all the sides of a triangle.
4. Generally, angle bisector doesn't intersect the opposite side perpendicularly.

→ The angle x is a right angle only in the case of an isosceles & equilateral triangle.

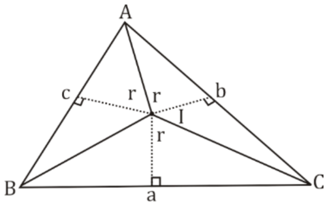


5. The angle between the external bisectors of two angles of a triangle is difference between right angle & half of the third angle

$\angle QSR = 90° - \frac{\angle P}{2}$



6. The ratio of area of triangle formed by incentre and three vertex are in ratio of their corresponding sides.



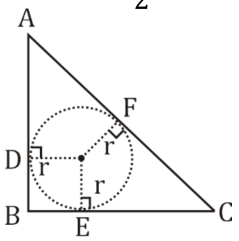
Ar ΔBIC : Ar ΔAIC : Ar ΔAIB = a : b : c

7. Area of any triangle is product of inradius and semi perimeter

A = r.s

8. Inradius of a right angle ΔABC

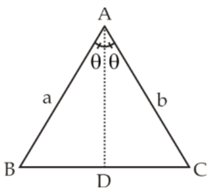
$$r = \frac{AB + BC - AC}{2}$$



9. Inradius of any other triangle

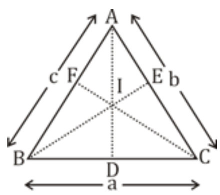
$$rs = \sqrt{s(s-a)(s-b)(s-c)}$$

10. Angle bisector in form of two adjacent sides and include angle.



$$AD = \frac{2bc\,cos\theta}{(b+c)}$$

11. Each angle bisector divided by incentre is divided in the ratio equal to the ratio of length of sum of two adjacent sides and opposite sides.

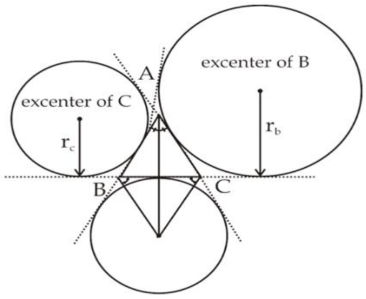


AI : ID = b+c : a

BI : IE = a+c : b

CI : IF = a+b : c

12. Excircle or escribed circle – Circle lying outside the triangle tangent to one of its sides and tangent to the extensions of the other two sides. There are three excircles of a triangle.
The centre of excircle i.e. excenter relative to a vertex is the intersection of the internal bisector of the vertex angle and the external bisectors of the other two vertex angle.



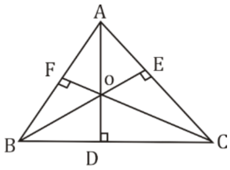
Exradii, $r_a = \sqrt{\frac{s(s-b)(s-c)}{s-a}}$, $r_s = \sqrt{\frac{s(s-a)(s-c)}{s-b}}$

$r_c = \sqrt{\frac{s(s-a)(s-b)}{s-c}}$

Area of $\Delta = \sqrt{r r_a r_b r_c}$

III. <u>ORTHOCENTRE</u>

→ It is the intersection point of all three altitudes of the triangle.



1. ∠BOC = 180° - ∠A, ∠AOC = 180° - ∠B, ∠AOB = 180° - ∠C
2. Orthocenter of ΔABC ⇒ O

Orthocenter of ΔBOC ⇒ A
Orthocenter of ΔAOB ⇒ C
Orthocenter of ΔAOC ⇒ B

3. Sum of sides > Sum of altitudes

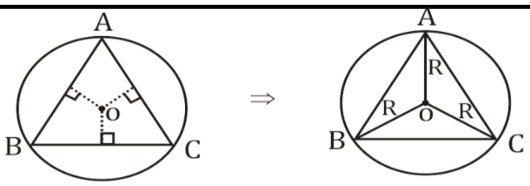
AB + BC + CA > AD + BE + CF

4.

Triangle	**Position of orthocenter**
1. Acute-angled triangle	Inside the triangle
2. Obtuse-angled triangle	Outside the triangle
3. Right-angled triangle	Right angle vertex

IV. <u>CIRCUMCENTRE</u>

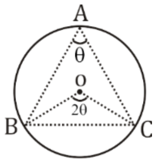
The points of intersection of the perpendicular bisector of the sides of a triangle.



R-circum-radius

Note:
Circumcenter is a point in the triangle which is equidistant from each vertex while incentre is a point which is equidistant from each side of the triangle.

1. The length from all 3 vertices to the circum-center is equal and is called circumradius.
2. ∠BOC = 2∠A
   ∠AOC = 2∠B, ∠AOB = 2∠C



This property can also be explained by property of a chord.

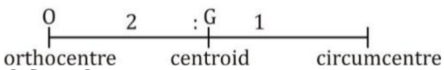
3.

Triangle	Circum-center
Obtuse-angled triangle	Outside the triangle
Right angled triangle	Mid-point of hypotenuse and circum-radius is half of hypotenuse

4. Circumradius of a triangle
   $R = \dfrac{abc}{4(\text{area of } \Delta)}$
5. The distance (d) between the circumcenter (r_c) and incentre (r_i) of a triangle is
   $d = \sqrt{r_c^2 - 2r_c r_i}$

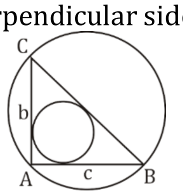
C. Mixed properties of centers of a triangle

1. In an equilateral triangle all the four centers are coincident i.e. centroid, incentre, circumcenter and orthocenter.
2. In any triangle orthocenter, centroid and circumcenter are collinear and centroid divides the join of orthocenter and circumcenter in the ratio 2 : 1.



$\dfrac{OG}{GC} = \dfrac{2}{1}$

3. The sum of diameters of circumcircle and incircle of right angled triangle is equal to the sum of its perpendicular sides.

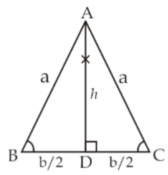


2(R+r) =b + c

H. TYPES OF TRIANGLES

Based upon sides	Based upon angles
1. Scalene triangle	1. Acute angle triangle
2. Isosceles triangle	2. Obtuse angle triangle
3. Equilateral triangle	3. Right angle triangle

1. Isosceles triangle



a. If Any two sides are equal.
AB = AC
Then, ∠C = ∠B
AD ⊥ BC
Then, BD = CD (AD = angle bisector)

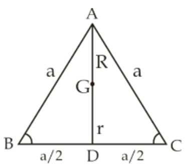
b. Height (Altitude), $AD = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{\frac{4a^2-b^2}{4}}$

c. $Area = \frac{1}{2} \times b \times \frac{\sqrt{4a^2-b^2}}{2} = \frac{b}{4}\sqrt{4a^2 - b^2}$

d. The length of perpendicular and median drawn by equal vertex to opposite sides are equal in length

e. The triangle formed by joining mid-point of three sides of an isosceles triangle is also an isosceles triangle.

2. Equilateral triangle



a. All sides and angles are equal.
⇒ AB = BC = CA = a
⇒ ∠A = ∠B = ∠C = 60°

b. Altitude, $AD, h = \frac{\sqrt{3}}{2}a$

c. All centers (orthocenter, circumcenter, incentre & centroid) lie on same point.

d. All medians = all altitudes = all perpendicular
Bisector = all angle bisector = $\frac{\sqrt{3}}{2}a$

e. Circumradius, $\boxed{R = \frac{a}{\sqrt{3}}}$

f. Inradius I = $\frac{\Delta}{s} \Rightarrow \boxed{r = \frac{a}{2\sqrt{3}}}$

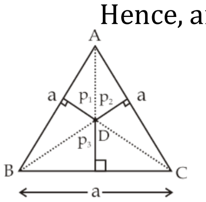
g. $\frac{R}{r} = \frac{2}{1}$

h. $\frac{\pi R^2}{\pi r^2} = \frac{area\ of\ circumcircle}{area\ of\ incircle} = \frac{4}{1}$

i. perimeter p = 3a then, semi-perimeter s = $\frac{3a}{2}$

j. $\boxed{area\ of\ \Delta = \frac{\sqrt{3}}{4}a^2}$ $\left(\because \Delta = \frac{1}{2} a \times a \times sin60°\right)$

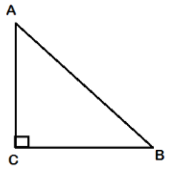
k. D is a point inside the equilateral triangle ABC. Three perpendiculars of length p1, p2 & p3 are drawn on sides AB, BC & AC of length 'a' respectively.
Hence, area of triangle ABC will be equal to the sum of all three triangles ADB, BDC & ADC.



$$a = \frac{2}{\sqrt{3}}[p_1 + p_2 + p_3]$$

3. Right angle triangle

One angle is equal to 90°



$\angle C = 90°$ then, $AB^2 = AC^2 + BC^2$

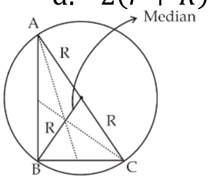
$c^2 = a^2 + b^2$ (This is Pythagoras theorem)

a. $\Delta = Area = \frac{1}{2}ab$

b. $R = \frac{abc}{4\Delta} = \frac{abc}{4.\frac{1}{2}ab} = \frac{c}{2} \Rightarrow \boxed{R = \frac{c}{2}}$

c. $r = \frac{a+b-c}{2}$

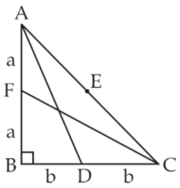
d. $2(r + R) = a + b$

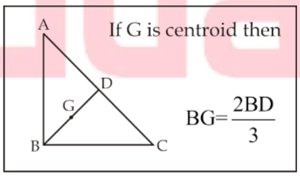


In right angle triangle shortest median = R(Circumradius)

e. Area of triangle ABC

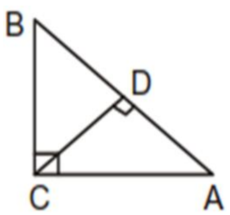
$\Delta = r.s = (s - c).s$





- $4(AD^2 + CF^2) = 5AC^2$
- $BD^2 = CD.AD$
- $BD = \frac{AB.BC}{AC}$

Some facts of Right-angle triangle



(A)	CD^2	=	$BD \times DA$
(B)	$BC \times CA$	=	$BA \times CD$
(C)	BC^2	=	$BD \times BA$
(D)	AC^2	=	$AD \times BA$
(E)	$\frac{BD}{DA}$	=	$\frac{BC^2}{AC^2}$
(F)	$\frac{1}{CD^2}$	=	$\frac{1}{BC^2} + \frac{1}{CA^2}$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