Quantitative Aptitude

Algebra (Identities)

Square formula	**1.** $(a + b)^2 = a^2 + b^2 + 2ab$ **2.** $(a - b)^2 = a^2 + b^2 - 2ab$ **3.** $(a + b)^2 = (a - b)^2 + 4ab$ **4.** $(a - b)^2 = (a + b)^2 - 4ab$ **5.** $(a^2 - ab + b^2)(a^2 + ab + b^2) = a^4 + a^2b^2 + b^2$ **6.** $(a + b)^2 - (a - b)^2 = 4ab$ **7.** $a^2 - b^2 = (a + b)(a - b)$
Cube formula	1. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ 2. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ 3. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ 4. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ 5. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ 6. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ **Special case 1**. If $a^2 - ab + b^2 = 0$ then $a^3 + b^3 = 0$ $\Rightarrow$ if $b = 1$, then $a^2-a+1=0$, then $a^3+1 = 0$ or $a^3 = -1$ **Special case 2**. If $a^2 + a + 1 = 0$ then $a^3 - 1 = 0$ or $a^3 = 1$ **Special case 3.** If $\frac{a}{b} + \frac{b}{a} = 1$ then $a^3 + b^3 = 0$ $\Rightarrow$ If $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$ then $a^3 + b^3 = 0$ **Special case 4.** If $\frac{a}{b} + \frac{b}{a} = -1$ then $a^3 - b^3 = 0$ $\Rightarrow$ If $\frac{a}{b} + \frac{b}{a} = \frac{1}{a+b}$ then $a^3 - b^3 = 0$ **Special case 5.** If $ab(a + b) = 1$ then $\frac{1}{a^3b^3} - a^3 - b^3 = 3$
Cube formulae of three variables	$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ (i) If $(a + b + c) = 0$ Then $a^3 + b^3 + c^3 - 3abc = 0$ $a^3 + b^3 + c^3 = 3abc$ (ii) If $a^3 + b^3 + c^3 - 3abc = 0$ a, b and c are distinct no then, $a + b + c = 0$ (iii) $a^3 + b^3 + c^3 - 3abc = 0$ a, b and c all are +ve integer no then, $a = b = c$. (iv) $a^2 + b^2 + c^2 - ab - bc - ca = 0$ $a^2 + b^2 + c^2 = ab + bc + ca$ then $a = b = c$
Square of three variable	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc\ \ ca)$
Power 2 identities	If $x - \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 + 2$ and $x^4 + \frac{1}{x^4} = (a^2 + 2)^2 - 2$
Power 3 identities	If $x + \frac{1}{x} = a$ then $x^3 + \frac{1}{x^3} = a^3 - 3a$ If $x - \frac{1}{x} = a$ then $x^3 - \frac{1}{x^3} = a^3 + 3a$

Power 5 identities	If $x + \frac{1}{x} = a$ Then $x^5 + \frac{1}{x^5} = (a^2 - 2)(a^3 - 3a) - a$ If $\left(x - \frac{1}{x}\right) = a$, then $x^5 - \frac{1}{x^5} = (a^2 + 2)(a^3 + 3a) - a$
Power 6 identities	$x + \frac{1}{x} = a$, then $x^6 + \frac{1}{x^6} = (a^3 - 3a)^2 - 2$
Power 7 identities	$x + \frac{1}{x} = a$, then $x^7 + \frac{1}{x^7} = ((a^2 - 2)^2 - 2) \times (a^3 - 3a) - a$ $x - \frac{1}{x} = a$, then $x^7 - \frac{1}{x^7} = ((a^2 + 2)^2 - 2) \times (a^3 + 3a) + a$
Other Important Identities	If $x + \frac{1}{x} = \sqrt{2}$ then $x^2 + \frac{1}{x^2} = 0$ or $x^4 + 1 = 0$ or $x^4 = -1$
	$x + \frac{1}{x} = \pm\sqrt{b^2 + 4}$ & $x - \frac{1}{x} = \pm\sqrt{a^2 - 4}$, Is also applicable of any power of x.
	If $x + \frac{1}{x} = \sqrt{3}$ then $x^3 + \frac{1}{x^3} = 0$ or $x^6 = -1$
	If $x + \frac{1}{x} = 2$ then x = 1 If $x + \frac{1}{x} = -2$ then x = -1
	If $x^n + \frac{1}{x^n} = a$ then $x^n - \frac{1}{x^n} = \pm\sqrt{a^2 - 4}$ If $x^n - \frac{1}{x^n} = b$ then $x^n - \frac{1}{x^n} = \pm\sqrt{b^2 + 4}$
	If x + y = 0, then x = -y Or x = 0, y = 0 ⇒if $x^2 + y^2 = 0$ then, $x^2 = 0$, x = 0 and, $y^2 = 0$, y = 0 ⇒if $(x - 1)^2 + (y - 2)^2 = 0$ then we can say x = 1 and y = 2
	Rationalising factor of the surd $x = \sqrt{a} \pm \sqrt{b}$ and $\frac{1}{x} = \sqrt{a} \mp \sqrt{b}$ Note : If xy = 1 then $\frac{1}{1+x} + \frac{1}{1+y} = 1$

Algebra (Quadratic Equation)

Concept	Formula / Definition
Quadratic Equation (Standard Form)	$ax^2 + bx + c = 0$ where a, b, c are real numbers and a ≠ 0.
Quadratic Polynomial	A polynomial of degree 2: $ax^2 + bx + c$.
Roots of Quadratic Equation	Values of x that satisfy $ax^2 + bx + c = 0$.
Discriminant (D)	$D = b^2 - 4ac$ Used to determine the nature of roots.
Nature of Roots	D > 0: Two distinct real roots D = 0: Two equal real roots D < 0: No real roots (Complex roots)
Quadratic Formula (Roots)	$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$

Sum of Roots (α + β)	-b / a
Product of Roots (αβ)	c / a
Factorization Method	Solve by expressing $ax^2 + bx + c = 0$ as $(px + q)(rx + s) = 0$.
Completing the Square Method	Solve by converting the equation into a perfect square trinomial.
Vertex Form of Quadratic Equation	$y = a(x - h)^2 + k$, where vertex is at (h, k).
Axis of Symmetry	x = -b / (2a)
Maximum/Minimum Value of Quadratic	y_max/min = -D / (4a) If a > 0: Minimum Value If a < 0: Maximum Value
Relation between Coefficients & Roots	Quadratic equation: a(x - α)(x - β) = 0 Expansion: $ax^2 - a(\alpha + \beta)x + a\alpha\beta$
Condition for Roots with Same Sign	Both roots have same sign if c > 0.
Condition for Roots with Opposite Signs	Roots have opposite signs if c < 0.
Reciprocal Roots Quadratic	If α, β are roots, then equation with reciprocal roots is $cx^2 + bx + a = 0$.
Quadratic Inequality	Solve $ax^2 + bx + c \geq 0$ or ≤ 0 using critical points (roots) and testing intervals.

Nature of Roots

- If $b^2 - 4ac = 0$ (Roots are real & equal)
- If $b^2 - 4ac > 0$ & a perfect square (Roots are real, unequal & rational)
- If $b^2 - 4ac > 0$ & not a perfect square (Roots are real, unequal & irrational)
- If $b^2 - 4ac < 0$ (Roots are imaginary)

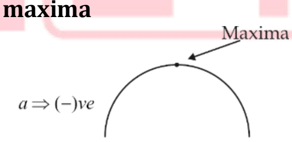
Algebra (Maxima and Minima, Polynomials, Linear Equalities)

Maximum and Minimum Value

Maximum and Minimum Value

1. **For quadratic expression $ax^2 + bx + c$**

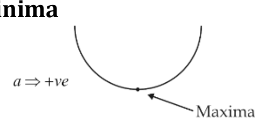
If a is – ve then graph has a maxima



for maximum value of expression, $x = \frac{b}{2a}$

Maximum value of expression $= \frac{4ac-b^2}{4a}$

if a is + ve then graph has a minima



for minimum value of expression $x = \frac{b}{2a}$

minimum value of expression $x = \frac{4ac-b^2}{4a}$

Note - Maximum value of a (function) + b(inverse function) minimum value is $2\sqrt{ab}$

$$ax^n + \frac{b}{x^n} \geq 2\sqrt{ab}$$

2. **Arithmetic Mean (AM) & Geometric Mean (GM)**	⇒A and b are two numbers Then, $A.M = \frac{a+b}{2}$ And $G.M. = \sqrt{ab}$ ⇒Thus, when a, b and c are three numbers Then, $A.M. = a + b + c$ $G.M. = \sqrt[3]{abc}$
Relation between A.M & G.M	⇒ A.M. ≥ G.M. When a, b, c are + ve Real number m and $\frac{1}{m}$ are two numbers $A.M. = \frac{m+\frac{1}{m}}{2}$, $G.M = \sqrt{m \times \frac{1}{m}}$ ⇒ A.M. ≥ G.M. $\frac{m+\frac{1}{m}}{2} \geq \sqrt[2]{m \times \frac{1}{m}}$ $\frac{m+\frac{1}{m}}{2} \geq 1$ $\frac{m+\frac{1}{m}}{2} \geq 2$ ⇒When m is +ve Real number. Minimum value of $m + \frac{1}{m} = 2$ And, ⇒We can say minimum value $m^n + \frac{1}{m^n} = 2$ When m is + ve real number **Note -** ⇒ If x + y will be given then, xy will be maximum, when x=y ⇒If x, y will be given then x + y will be minimum when x = y here x & y (+ve) real number

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